CHAPTER 1

Motion Matters

1.1 MOTION

One of the most important properties of the objects that make up our physical world is the fact that they can move. Motion is all around us, from falling leaves and tumbling rocks, to moving people and speeding cars, to jet planes, orbiting space satellites, and planets. Understanding what motion is, how it can be described, and why it occurs, or doesn't occur, are therefore essential to understanding the nature of the physical world. You saw in the Prologue that Plato and others argued that mathematics can be used as a tool for comprehending the basic principles of nature. You also saw that we can use this tool to great advantage when we apply it to precise observations and experiments. This chapter shows how these two features of modern physics—mathematics and experiment—work together in helping us to understand the thing we call motion.

Motion might appear easy to understand, but initially it's not. For all of the sophistication and insights of all of the advanced cultures of the past, a really fundamental understanding of motion first arose in the scientific "backwater" of Europe in the seventeenth century. Yet that backwater was experiencing what we now know as the Scientific Revolution, the "revolving" to a new science, the science of today. But it wasn't easy. At that time it took some of the most brilliant scientists entire lifetimes to comprehend motion. One of those scientists was Galileo Galilei, the one whose insights helped incorporate motion in modern physics.

1.2 GALILEO

Galileo Galilei was born in Pisa in 1564, the year of Michelangelo's death and Shakespeare's birth. Galileo (usually called by his first name) was the son of a noble family from Florence, and he acquired his father's active interest in poetry, music, and the classics. His scientific inventiveness also began to show itself early. For example, as a medical student, he constructed a simple pendulum-type timing device for the accurate measurement of pulse rates. He died in 1642 under house arrest, in the same year as Newton's birth. The confinement was the sentence he received after being convicted of heresy by the high court of the Vatican for advocating the view that the Earth is not stationary at the center of the Universe, but instead rotates on its axis and orbits the Sun. We'll discuss this topic and the results later in Chapter 2, Section 12.

FIGURE 1.1 Galileo Galilei (1564–1642).

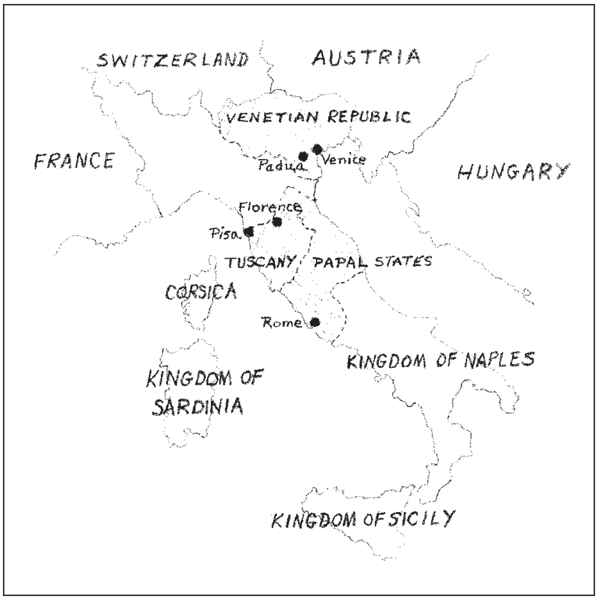


FIGURE 1.2 Italy, ca. 1600 (shaded portion).

After reading the classical Greek philosopher–scientists Euclid, Plato, and Archimedes, Galileo changed his interest from medicine to physical science. He quickly became known for his unusual scientific ability. At the age of 26 he was appointed Professor of Mathematics at Pisa. There he showed an independence of spirit, as well as a lack of tact and patience. Soon after his appointment he began to challenge the opinions of the older professors, many of whom became his enemies and helped convict him later of heresy. He left Pisa before completing his term as professor, apparently forced out by financial difficulties and his enraged opponents. Later, at Padua in the Republic of Venice, Galileo began his work on astronomy, which resulted in his strong support of our current view that the Earth rotates on its axis while orbiting around the Sun.

A generous offer of the Grand Duke of Tuscany, who had made a fortune in the newly thriving commerce of the early Renaissance, drew Galileo back to his native Tuscany, to the city of Florence, in 1610. He became Court Mathematician and Philosopher to the Grand Duke, whose generous patronage of the arts and sciences made Florence a leading cultural center of the Italian Renaissance, and one of the world's premier locations of Renaissance art to this day. From 1610 until his death at the age of 78, Galileo continued his research, teaching, and writing, despite illnesses, family troubles, and official condemnation.

Galileo's early writings were concerned with *mechanics*, the study of the nature and causes of the motion of matter. His writings followed the stan-


DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue scienze
Attenenti alla
MECANICA & i MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filosofo e Matematico primario del Serenissimo
Grand Duca di Toscana.
Con vna Appendice del centro di grauità d'alcuni Solidi.
IN LEIDA,
Appresso gli Elsevirii. M. D. C. XXXVIII.

FIGURE 1.3 Title page from Galileo's *Discourses and Mathematical Demonstrations Concerning Two New Sciences Pertaining to Mechanics and Local Motion* (1638).

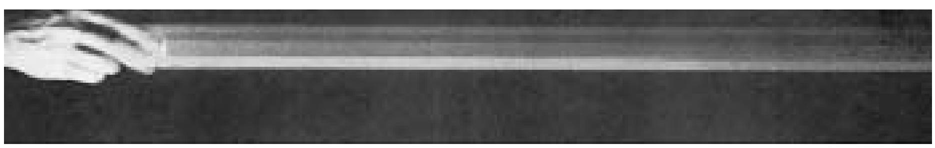
dard theories of his day, but they also showed his awareness of the shortcomings of those theories. During his mature years his chief interest was in astronomy. However, forbidden to teach astronomy after his conviction for heresy, Galileo decided to concentrate instead on the sciences of mechanics and hydrodynamics. This work led to his book *Discourses and Mathematical Demonstrations Concerning Two New Sciences* (1638), usually referred to either as the *Discorsi* or as *Two New Sciences*. Despite Galileo's avoidance of astronomy, this book signaled the beginning of the end of Aristotle's cosmology and the birth of modern physics. We owe to Galileo many of the first insights into the topics in the following sections.

1.3 A MOVING OBJECT

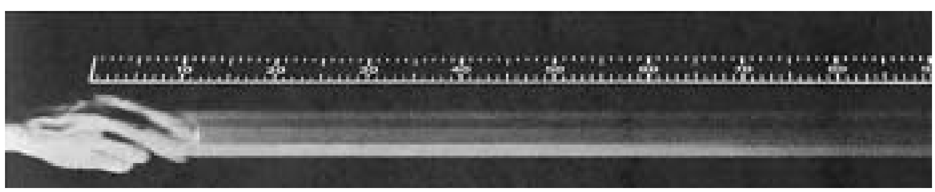
Of all of the swirling, whirling, rolling, vibrating objects in this world of ours, let's look carefully at just one simple moving object and try to describe its motion. It's not easy to find an object that moves in a simple way, since most objects go through a complex set of motions and are subject to various pushes and pulls that complicate the motion even further.

Let's watch a dry-ice disk or a hockey puck moving on a horizontal, flat surface, as smooth and frictionless as possible. We chose this arrangement so that friction at least is nearly eliminated. Friction is a force that will impede or alter the motion. By eliminating it as much as possible so that we can generally ignore its effects in our observations, we can eliminate one complicating factor in our observation of the motion of the puck. Your instructor may demonstrate nearly frictionless motion in class, using a disk or a cart or some other uniformly moving object. You may also have an opportunity to try this in the laboratory.

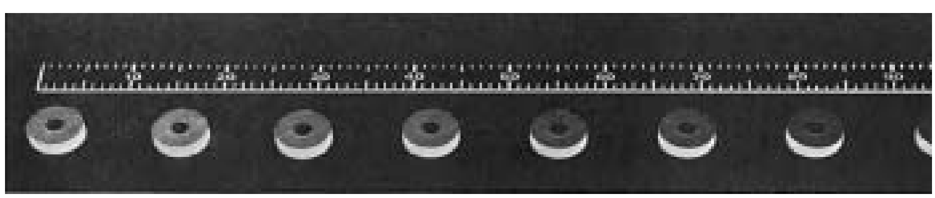
If we give the frictionless disk a push, of course it moves forward for a while until someone stops it, or it reaches the end of the surface. Looking just at the motion before any remaining friction or anything else has a noticeable effect, we photographed the motion of a moving disk using a camera with the shutter left open. The result is shown in Figure 1.4. As nearly as you can judge by placing a straight edge on the photograph, the disk moved in a straight line. This is a very useful result. But can you tell anything further? Did the disk move steadily during this phase of the motion, or did it slow down? You really can't tell from the continuous blur. In order to answer, we have to improve the observation by controlling it more. In other words, we have to experiment.



(*a*)



(*b*)



(*c*)

FIGURE 1.4 Time exposures of disk set in motion.

It would be helpful to know where the disk is at various times. In the next photograph, Figure 1.4b, we put a meter stick on the table parallel to the expected path of the disk, and then repeated the experiment with the camera shutter held open.

This photograph tells us again that the disk moved in a straight line, but it doesn't tell us much more. Again we have to improve the experiment. In this experiment the camera shutter will be left open and everything else will be the same as last time, except that the only source of light in the darkened room will come from a stroboscopic lamp. This lamp produces bright flashes of light at short time intervals which we can set as we please. We set each flash of light to occur every tenth of a second. (Each flash is so fast, one-millionth of a second, that its duration is negligible compared to one-tenth of a second.) The result is shown in Figure 1.4c.

This time the moving disk is seen in a series of separate, sharp exposures, or "snapshots," rather than as a continuous blur. Now we can actually see some of the positions of the front edge of the disk against the scale of the meter stick. We can also determine the moment when the disk was at each position from the number of strobe flashes corresponding to each position, each flash representing one-tenth of a second. This provides us with some very important information: *we can see that for every position reading of the disk recorded on the film there is a specific time, and for every time there is a specific position reading.*

Now that we know the position readings that correspond to each time (and vice versa), we can attempt to see if there is some relationship between them. This is what scientists often try to do: study events in an attempt to see patterns and relationships in nature, and then attempt to account for them using basic concepts and principles. In order to make the discussion a little easier, scientists usually substitute symbols at this point for different measurements as a type of shorthand. This shorthand is also very useful, since the symbols here and many times later will be found to follow the "language" of mathematics. In other words, just as Plato had argued centuries earlier, our manipulations of these basic symbols according to the rules of mathematics are expected to correspond to the actual behavior of the related concepts in real life. This was one of the great discoveries of the scientific revolution, although it had its roots in ideas going back to Plato and the Pythagoreans. You will see throughout this course how helpful mathematics can be in understanding actual observations.

In the following we will use the symbol d for the *position reading* of the front edge of the disk, measured from the starting point of the ruler, and the symbol t for the amount of *elapsed time* from the start of the experiment that goes with each position reading. We will also use the standard abbreviations cm for centimeters and s for seconds. You can obtain the values of some pairs of position readings d and the corresponding time readings t directly from the photograph. Here are some of the results:

Position	d *(in centimeters)*	t *(in seconds)*
1	6.0	0.1
2	19.0	0.2
3	32.0	0.3
4	45.0	0.4
5	58.0	0.5
6	71.0	0.6
7	84.0	0.7

From this table you can see that in each case the elapsed time increased by one-tenth of a second from one position to the next—which is of course what we expect, since the light flash was set to occur every one-tenth of a second. We call the duration between each pair of measurements the "time interval." In this case the time intervals are all the same, 0.1 s. The distance the disk traveled during each time interval we call simply the "distance traveled" during the time interval.

The time intervals and the corresponding distances traveled also have special symbols, which are again a type of shorthand for the concepts they represent. The time interval between any two time measurements is given the symbol Δt. The distance traveled between any two position readings is given the symbol Δd. These measurements do not have to be next to each other, or successive. They can extend over several flashes or over the entire motion, if you wish. The symbol Δ here is the fourth letter in the Greek alphabet and is called "delta." Whenever Δ precedes another symbol, it means "the change in" that measurement. Thus, Δd does *not* mean "Δ multiplied by d." Rather, it means "the change in d" or "the distance traveled." Likewise, Δt stands for "the change in t" or "the time interval." Since the value for Δt or Δd involves a change, we can obtain a value for the amount of change by subtracting the value of d or t at the start of the interval from the value of d or t at the end—in other words, how much the value is at the end minus the value at the start. In symbols:

$$\Delta d = d_{\text{final}} - d_{\text{initial}},$$

$$\Delta t = t_{\text{final}} - t_{\text{initial}}.$$

The result of each subtraction gives you the difference or the change in the reading. That is why the result of subtraction is often called the "difference."

Now let's go back and look more closely at our values in the table for the position and time readings for the moving disk. Look at the first time interval, from 0.1 s to 0.2 s. What is the value for Δt? Following the above definition, it is $t_{\text{final}} - t_{\text{initial}}$, or in this case 0.2 s − 0.1 s, which is 0.1 s. What is the corresponding change in the position readings, Δd? In that time interval the disk's position changed from 6.0 cm to 19.0 cm. Hence, the value for Δd is 19.0 cm − 6.0 cm, which is 13.0 cm.

What would you expect to find for Δd if the disk had been moving a little faster? Would Δd be larger or smaller? . . . If you answered larger, you're right, since it would cover more ground in the same amount of time if it's moving faster. What would happen if it was moving slower? . . . This time Δd would be smaller, since the disk would cover less ground in the given amount of time. So it seems that one way of describing how fast or how slow the disk is moving is to look at how far it travels in a given time interval, which is called the "rate" that the distance changes.

Of course, we could also describe how fast it goes by how much time it takes to cover a certain fixed amount of distance. Scientists in the seventeenth century made the decision not to use this definition, but to use the first definition involving the distance traveled per time interval (rather than the reverse). This gives us the "rate" of motion, which we call the *speed*. (The idea of *rate* can apply to the growth or change in anything over time, not just distance; for example, the rate at which a baby gains weight, or the rate of growth of a tomato plant.)

We can express the rate of motion—the speed—as a *ratio*. A ratio compares one quantity to another. In this case, we are comparing the amount of distance traveled, which is represented by Δd, to the size of the time interval, which is represented by Δt. Another way of saying this is the amount of Δd per Δt. If one quantity is compared, or "per," another amount, this can be written as a fraction

$$\text{speed} = \frac{\Delta d}{\Delta t}.$$

In words, this says that the speed of an object during the time interval Δt is the ratio of the distance traveled, Δd, to the time interval, Δt.

This definition of the speed of an object also tells us more about the meaning of a ratio. A ratio is simply a fraction, and speed is a ratio with the distance in the numerator and time in the denominator. As you know, a fraction always means division: in this case, the rate of motion or the speed given by distance traveled, Δd, *divided by* the time interval, Δt.

There is still one small complication: we don't know exactly what the disk is doing when we don't see it between flashes of the light. Probably it is not doing anything much different than when we do see it. But due to

■ NOW YOU TRY IT

The Tour de France is a grueling test of endurance over the varied terrain of France. The total distance of the bicycle race is 3664 km (2290 mi). Lance Armstrong, cancer survivor and winner of the 1999 Tour de France, set a new record in covering this distance in 91.1 hr of actual pedaling. The race included breaks each night along the way. From the data given, what was Armstrong's average speed for the entire race while he was riding? (This speed broke the old record for the course of 39.9 km/hr.) He repeated his win in 2000.

FIGURE 1.5 Lance Armstrong.

friction it may have slowed down just a bit between flashes. It could also have speeded up a bit after being hit by a sudden blast of air; or perhaps nothing changed at all, and it kept right on moving at the exact same rate. Since we don't know for certain, the ratio of Δd to Δt gives us only an "average," because it assumes that the rate of increase of d has not changed at all during the time interval Δt. This is another way of getting an average of similar numbers, rather than adding up a string of values and dividing by the number of values. We give this ratio of Δd to Δt a special name. We call it the *average speed* of the disk in the time interval Δt. This also has a special symbol, v_{av}:

$$\frac{\Delta d}{\Delta t} = v_{av}.$$

These symbols say in words: *The measured change in the position of an object divided by the measured time interval over which the change occurred is called the average speed.*

This *definition* of the term *average speed* is useful throughout all sciences, from physics to astronomy, geology, and biology.

To see how all this works, suppose you live 20 mi from school and it takes you one-half hour to travel from home to school. What is your average speed?

Answer: The distance traveled Δd is 20 mi. The time interval Δt is 0.5 hr. So the average speed is

$$v_{\mathrm{av}} = \frac{\Delta d}{\Delta t} = \frac{20\ \mathrm{mi}}{0.5\ \mathrm{hr}} = 40\ \frac{\mathrm{mi}}{\mathrm{hr}}.$$

Did you actually travel at a steady speed of 40 mi/hr for the entire half-hour? Probably not. There were probably stop lights, slow traffic, corners to turn, and stretches of open road. In other words, you were constantly speeding up and slowing down, even stopping, but you averaged 40 mi/hr for the trip. Average speed is a handy concept—even though it is possible that very rarely during your travel you went at exactly 40 mi/hr for any length of the road.

Back to the Moving Disk

Let's go back to the disk to apply these definitions. (See the table on page 21.) What is the average speed of the disk in the first time interval? Substituting the numbers into the formula that defines average speed

$$v_{\mathrm{av}} = \frac{\Delta d}{\Delta t} = \frac{13.0\ \mathrm{cm}}{0.1\ \mathrm{s}} = 130\ \frac{\mathrm{cm}}{\mathrm{s}}.$$

What about the next interval, 0.2 s to 0.3 s? Again, Δt is 0.1 s and Δd is 32.0 cm − 19.0 cm = 13.0 cm. So,

$$v_{\mathrm{av}} = \frac{\Delta d}{\Delta t} = \frac{13.0\ \mathrm{cm}}{0.1\ \mathrm{s}} = 130\ \mathrm{cm/s}.$$

The average speed is the same. Notice again that in finding the change in d or in t, we always subtract the beginning value *from* the ending value.

We don't have to consider only successive time readings. Let's try a larger time interval, say from 0.2 s to 0.7 s. The time interval is now $\Delta t = 0.7\ \mathrm{s} - 0.2\ \mathrm{s} = 0.5\ \mathrm{s}$. The corresponding distance traveled is $\Delta d = 84.0\ \mathrm{cm} - 19.0\ \mathrm{cm} = 65.0\ \mathrm{cm}$. So the average speed is

$$v_{\mathrm{av}} = \frac{\Delta d}{\Delta t} = \frac{65.0\ \mathrm{cm}}{0.5\ \mathrm{s}} = 130\ \mathrm{cm/s}.$$

What can you conclude from all of these average speeds? Our data indicate that the disk maintained the same average speed throughout the entire experiment as recorded on the photograph (we don't know what it did before or after the photograph was made). We say that anything that moves at a constant speed over an interval of time has a *uniform speed.*

1.4 PICTURING MOTION

Most sports involve motion of some sort. Some sports, such as swimming, jogging, bicycling, ice skating, and roller blading, involve maintaining speed over a given course. If it's a race, the winner is of course the person who can cover the course distance in the shortest time, which means the fastest average speed. Here the word "average" is obviously important, since no swimmer or biker or runner moves at a precisely uniform speed.

Let's look at an example. Jennifer is training for a running match. Recently, she made a trial run. The course was carefully measured to be 5000 m (5 km, or about 3.1 mi) over a flat road. She ran the entire course in 22 min and 20 s, which in decimal notation is 22.33 min. *What was her average speed in kilometers per minute during this run?*

FIGURE 1.6

The distance traveled Δd was 5 km; the time interval Δt was 22.33 min. So her average speed was

$$v_{\text{av}} = \frac{\Delta d}{\Delta t} = \frac{5.00 \text{ km}}{22.33 \text{ min}} = 0.224 \text{ km/min}$$

Did Jennifer actually run the 5 km at the constant speed of 0.224 km/min? Very likely not. To find out more about her average speed on different parts of the road, let's compare her overall average speed with her average speed during different segments of the run, just as we did earlier with the disk—only this time the distance covered (Δd) will be fixed, rather than having a fixed time interval Δt. To do this, we stationed five observers at 1-km intervals along the course. Δd will be the distance between neighboring observers. Each had a stopwatch that started at the start of her run. As Jennifer passed each position, the observer there read on the stopwatch the elapsed time from the start and recorded the results. The results, in decimals, are shown below. Take a moment to study this table and try to understand what each column refers to and how these numbers were obtained.

d *(in km)*	t *(in min)*	Δd *(km)*	Δt *(min)*	v_{av} *(km/min)*
0.00	0.00	—	—	—
1.00	4.40	1.00	4.40	0.227
2.00	8.83	1.00	4.43	0.226
3.00	13.40	1.00	4.57	0.219
4.00	18.05	1.00	4.65	0.215
5.00	22.33	1.00	4.28	0.234

Once you understand these data you can, for instance, determine Jennifer's average speed for the first kilometer and for the last kilometer separately:

average speed for the first kilometer is

$$\frac{\Delta d}{\Delta t} = \frac{1.0 \text{ km}}{4.40 \text{ min} - 0.00 \text{ min}}$$
$$= 0.227 \text{ km/min},$$

average speed for the last kilometer is

$$\frac{\Delta d}{\Delta t} = \frac{1.0 \text{ km}}{22.33 \text{ min} - 18.05 \text{ min}}$$
$$= 0.234 \text{ km/min}.$$

The results for v_{av} for each 1-km distance are given in the table on page 26, rounded off to the third decimal place. As you can see, none of these average speeds for each kilometer distance turned out to be exactly the overall average, 0.224 km/min. You can also see that Jennifer varied her speed during the run. In fact, on the average she slowed down steadily from the first to the fourth kilometer, then she speeded up dramatically as she approached the end. That last kilometer was the fastest of the five. In fact, it was faster than the overall average, while the intermediate length segments were covered more slowly.

Another way of observing Jennifer's run—a way favored by athletic trainers—is not just to look at the position and time readings in a table, but to look at the pattern these pairs of numbers form in a picture or "graphical representation" of the motion, called a "graph." By placing the position readings along one axis of a sheet of graph paper and the corresponding time readings along the other axis, each pair of numbers has a unique place on the graph, and together all the pairs of numbers form an overall pattern that gives us a picture of what happened during the overall motion. Usually the time is placed on the horizontal axis or x-axis, because as time increases to the right we think of the pattern as "progressing" over time. (Some common examples might be daily temperature data for a period of time, or sales of products by quarters for the past years.)

Two graphs of Jennifer's run are shown in Figures 1.7 and 1.8. The first shows the labeled axes and the data points. The second shows line segments connecting each pair of dots and the "origin," which is the point $d = 0$, $t = 0$ in the lower left corner, corresponding to the start of the run.

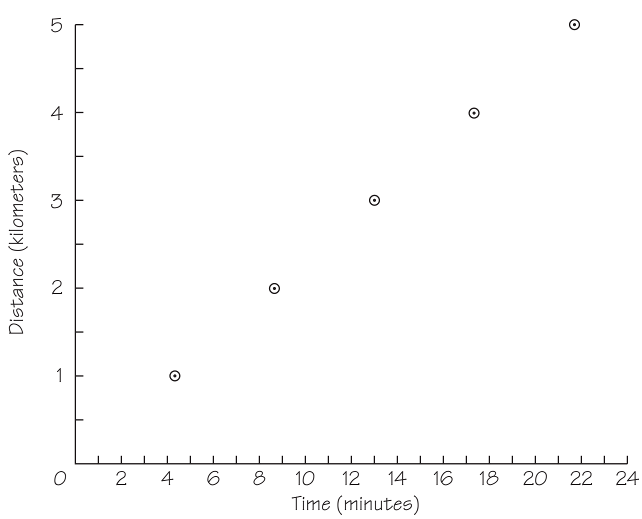


FIGURE 1.7 Distance vs. Time graph.

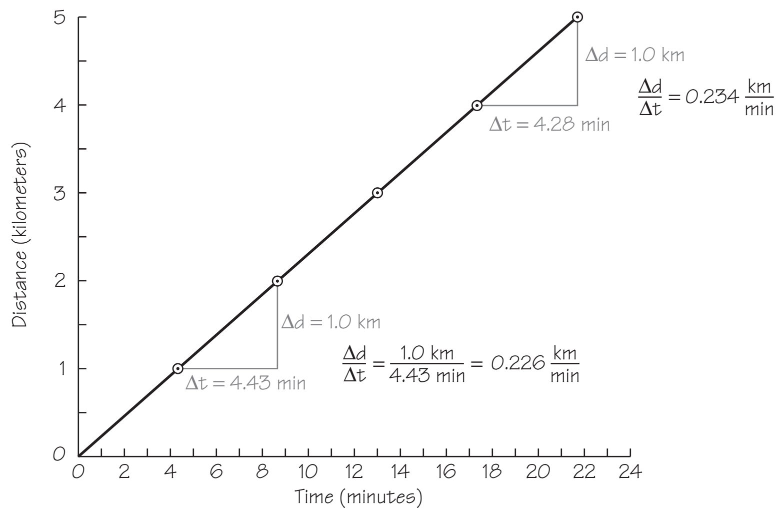


FIGURE 1.8 Distance vs. Time graph with slopes.

What can you observe, or "read," from the second graph? First of all, you can see (by putting a straight edge to it) that the first line segment is fairly steep, but that the last one is the steepest of all. There was a comparatively short time interval for the first leg of 1 km, and an even shorter one for the last leg of 1 km in length. In other words, Jennifer got off to a relatively fast start during the first kilometer distance; but she went even faster during the last one. In between, the steepness of each line declines, indicating that it took her longer and longer to cover the same distance of 1 km. She was slowing down. You can also obtain this result from the fourth column in the table (Δt). Notice how, for each kilometer covered, the time intervals Δt increase in the middle, but are less for the first and last kilometers. The distance covered stays the same, but the time to do so varies. This is seen as a change in average speed for each kilometer, as you can see from the last column of the table, and this agrees with the changing steepness of each line segment. We can conclude from this: *The steepness of each line segment on the graph is an indication of the average speed that Jennifer was moving in that interval*. The faster she ran, the steeper the line segment. The slower she ran, the less steep the line segment.

Looked at in this way, a graph of distance readings plotted on the y-axis against the corresponding time readings on the x-axis provides us with a visual representation of the motion, including the *qualitative* variations in the speed. But this kind of representation does not tell us directly what the *quantitative* speed was at any particular moment, what we can think of as the actual momentary speed in kilometers per minute, similar to the information on the speedometer of a moving car. We will come back to this notion later, but for now we will look only at the average speed in each time interval. Since the steepness of the graph line is an indication of the average speed during that segment of the graph, we need a way of measuring the steepness of a line on a graph. Here we must turn again to mathematics for help, as we often will.

The steepness of a graph line at any point is related to the change in vertical direction (Δy) during the corresponding change in horizontal direction (Δx). By definition, the ratio of these two changes is called the "slope":

$$\boxed{\frac{\Delta y}{\Delta x} = \text{slope of a line.}}$$

Slope can be used to indicate the steepness of a line in any graph. In a distance–time graph, like the one for Jennifer's run, the distance from the start is plotted on the vertical axis (d in place of y) and the corresponding times are plotted on the horizontal axis (t in place of x). Together, d and t make a series of points forming a line in the plane of the graph. In such a graph, the slope of a straight line representing the motion of a person or an object is defined as follows:

$$\text{slope} = \frac{\Delta d}{\Delta t}.$$

Does this ratio look familiar? It reminds us of the definition of average speed, $v_{av} = \Delta d/\Delta t$. The fact that these two definitions are identical means

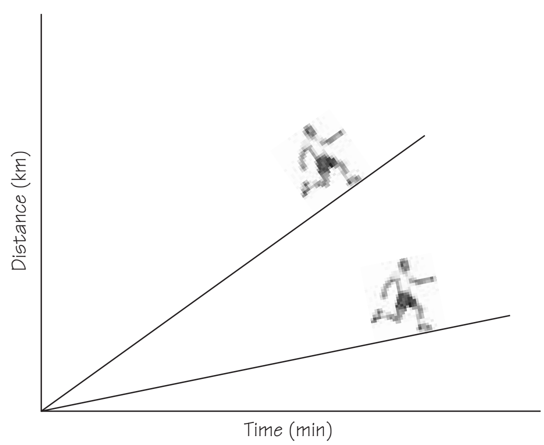


FIGURE 1.9 Distance vs. Time graphs for runners at two different speeds. Which one is faster?

that v_{av} is numerically equal to the slope on distance–time graphs! In other words, *the slope of any straight-line part of a graph of distance versus time gives a measure of the average speed of the object during that time interval.*

In short, we can use simple geometry to "capture" an observed motion.

1.5 SPEED AND VELOCITY

You may wonder why we used the letter v instead of s for speed. The reason is that the concept of speed is closely related to the concept of *velocity*, from which the symbol v arises. However, these two are *not* the same when used in precise technical terms. The term "velocity" is used to indicate *speed in a specified direction*, such as 50 km/hr to the north or 130 cm/s to the right. Speed indicates how fast something is moving regardless of whether or how it may change in direction. But velocity indicates how fast it is moving *and* the direction it is moving. In physics, anything that has both a size or magnitude and a direction is called a *vector.* Since a vector points in a definite direction, it is usually presented by an arrow in diagrams, such as the one in Figure 1.10. The direction of the arrow indicates the direction of the velocity. The length of the arrow indicates its magnitude, the speed.

Here is an example: one car is traveling west on a road at 40 mi/hr. Another identical car is traveling east on the same road at 40 mi/hr. Is there any difference between the motions of these two cars? Both are going at 40 mi/hr, so there is no difference in their speeds. The only difference is that one car is going west and the other is going east. That difference is obviously important if the two cars happen to be traveling on the same side of the road and meet! Obviously, the direction of the motion of each car, as well as its speed, is important in describing what happens in situations such as this. That is why it is necessary to use the velocity vector, which incorporates both speed and direction, in situations where both of these factors are important. When the direction of the motion is not important, the scalar speed is usually sufficient.

The velocity vector also has a special symbol. Following standard practice in textbooks, we will represent the velocity, or any vector, in bold font.

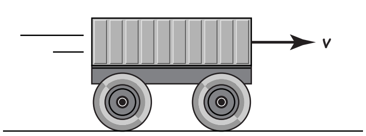


FIGURE 1.10 Cart traveling at velocity **v**.

So the symbol for the velocity vector is **v**. You will see other examples of vectors in this and later chapters.

When a direction is not specified, and only the size, or magnitude, is of interest, we'll remove the bold font and just use the italic letter v, which is the speed. This symbol represents only the *magnitude* of the velocity—how fast it is regardless of the direction. Speed is an example of what is known in technical terms as a *scalar*. It has no direction, just a reading on some scale. Some other examples of scalars are mass, temperature, and time.

1.6 CHANGING THE SPEED

In the previous sections we looked at two examples of motion. One was a disk moving, as far as we could tell, at constant or uniform speed; the other was a runner whose speed clearly varied as she went over a given distance. There are many examples in nature of moving objects that undergo changes of speed and/or direction. As you walk to class, you may realize you are late and pick up your pace. An airplane landing at an airport must decrease its altitude and slow its speed as it lands and comes to a halt on the runway. Cars going around a curve on a freeway usually maintain their speed but change the direction of their motion. A growing tomato plant may spurt up over a number of days, then slow its growth in height as it starts to bud. Many other such examples come to mind.

Motions that involve changes in speed and/or direction over a period of time are obviously an important part of the motions that occur in nature. A change in the velocity of a moving object, during an interval of time, is known as the *acceleration*. Since an interval of time is the amount of time that elapses from one instant to another, we need to know the velocity of the object at each instant of time, in order to find the acceleration. How can we examine changes of speed from one instant to another? So far, we have talked only about *average* speeds over a time interval. One way to study changes of speed from one instant to another is to take advantage of a modern device that measures speed at any given time for us by converting the speed of, say, the wheels of a car into a magnetic force that turns a needle on a dial. This is the basic principle behind the speedometer. Although this refers only to the motion of a car, the general principles will be the same for other moving objects.

The speedometer in Figure 1.11 reads in kilometers per hour. This car was driven on an open highway in a straight line, so no changes of direction occurred. As the car was traveling on the road, snapshots were made of the speedometer reading every second for 10 s after an arbitrary start, where we

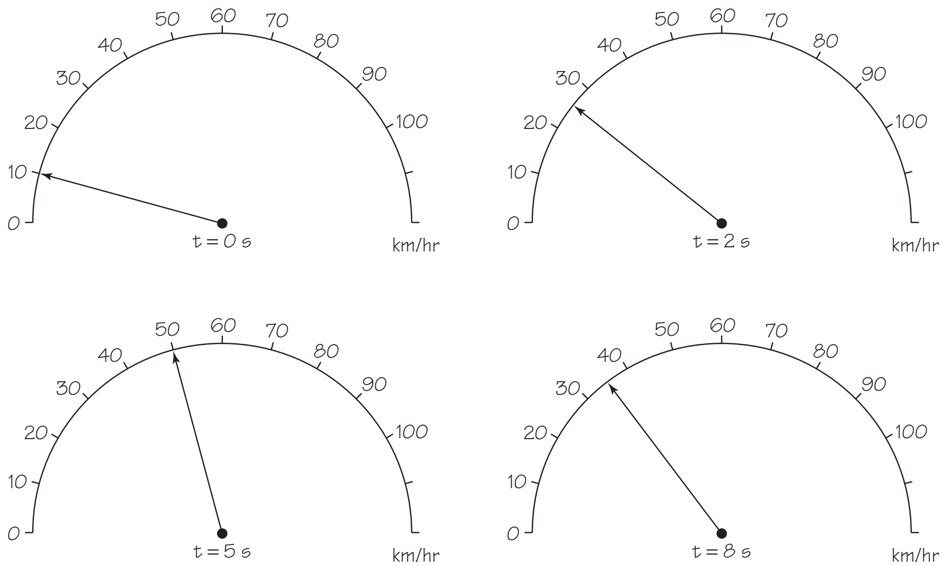


FIGURE 1.11 Speedometer snapshots. t = 0 s at an arbitrary point when car already in motion.

set $t = 0$. Some of the snapshots are shown below. Similarly to the earlier distance and time readings for the disk and runner, this experiment provides us with ten pairs of speed and time readings for the motion of this car. We have no data or snapshots for the motion of the car before or after this experiment. The results are given in the table below:

v (km/hr)	*t (s)*
10	0
18	1
26	2
34	3
42	4
50	5
50	6
50	7
35	8
20	9
5	10

We have to determine first what these numbers really mean before we can analyze what the car was actually doing. This is a common procedure in scientific research: determine first the precise meaning of the data be-

The instantaneous speed may be defined as the limit, as the time interval approaches zero, of the ratio of distance traveled per time interval, or:

$$v = \lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t}.$$

fore attempting to analyze them. As you know from the previous sections, the average speed is defined as the ratio of the distance traveled to the time interval. In this case, the speedometer mechanism used very small increments of time to obtain these speeds. The increments of time and the corresponding distances the car traveled are so small that, within the limits of precision of this experiment, we can assume that for all practical purposes they are zero.

So we can say that, within the limits of accuracy of this experiment, the speeds shown in the table on page 32 indicate the "instantaneous speed" of the car, and the speed at the instant of time shown in the second column. Of course, with faster photographic equipment or a faster speedometer mechanism, we might see a slightly different instantaneous speed, but that would be beyond the limits of accuracy of *this* experiment.

Now we are ready to interpret what this car was actually doing, according to the data that we have. Similarly to the case of distance and time for the disk or runner, we can look at the change in speed in each time interval, which will help us to see how the motion is changing. Let's add three more columns to the table. The first will be the change Δv in the instantaneous speeds measured by the speedometer; the second will be the time interval, Δt, as before; and the third will be their ratio, $\Delta v/\Delta t$.

v (km/hr)	t (s)	Δv (km/hr)	Δt (s)	$\Delta v/\Delta t$ (km/hr/s)
10	0	—	—	—
18	1	8.0	1.0	8.0
26	2	8.0	1.0	8.0
34	3	8.0	1.0	8.0
42	4	8.0	1.0	8.0
50	5	8.0	1.0	8.0
50	6	0.0	1.0	0.0
50	7	0.0	1.0	0.0
35	8	−15.0	1.0	−15.0
20	9	−15.0	1.0	−15.0
5	10	−15.0	1.0	−15.0

At first glance this looks really puzzling! What was this car doing? To find out, let's "analyze" the motion, that is, let's take a closer look at the motion piece by piece. Starting at the top (when the experiment began) at time 0, the car was already going at 10 km/hr. From 0 to 5 s, the speed increased by an additional 8.0 km/hr in each time interval of 1.0 s (see the third column). As with distance and time, the ratio of Δv to Δt gives us a quantitative value for the average rate of change of the speed in each time interval. As you can see from the last column, during the first 5 s the speed of the car increased at the average rate of 8.0 km/hr/s, that is, 8.0 km/hr in each second.

Since it is so important in describing changing speeds, the ratio $\Delta v/\Delta t$ also has been given a special name. It is called the *average acceleration* in each time interval. This has the symbol a_{av}:

$$\boxed{\frac{\Delta v}{\Delta t} = a_{av}.}$$

These symbols say in words: *The change in the instantaneous speed of an object divided by the time interval over which the change occurs is defined as the average acceleration of the object.* In this case, the car maintained a constant average acceleration for the first 5 s. This behavior is called "uniform acceleration."

We have to add just one more idea. Since the speeds above were all in the same direction, we didn't need to refer to the velocity. But sometimes the direction of motion does change, as when a car goes around a corner, even though the speed stays the same. We can expand our definition of acceleration to include changes of direction as well as changes of speed by simply replacing the speed v in the above formula by the velocity vector, $\mathbf{v}$. Since velocity is a vector, the acceleration is also a vector, so we have the following definition:

$$\boxed{\frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{a}_{av}.}$$

These symbols say in words: *The change in the velocity of an object divided by the time interval over which the change occurs is defined as the average acceleration vector.* In the example so far, we can say that the car velocity increased uniformly in the forward direction, so it maintained a uniform average acceleration vector of 8 km/hr/s in the forward direction for the first 5 s.

Now let's go on to the sixth and seventh seconds. What is the car doing? The change in the speed is zero, so the ratio of $\Delta v/\Delta t$ gives zero, while the direction remains unchanged. Does that mean the car stopped? No, what it means is that the *acceleration* (not the speed) stopped; in other words, the car stopped changing its speed for 2 s, so the average speed (and average velocity) remained constant. The car cruised for 2 s at 50 km/hr.

Now what happened during the last 3 s? You can figure this out yourself . . . but, here's the answer: the car is moving with *negative* accel-

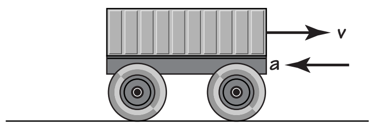


FIGURE 1.12 Cart traveling at v with acceleration opposite.

eration. (This is sometimes called the "deceleration.") Probably the driver put on the brakes, slowing the car down gradually to 5 km/hr over 3 s. How do we know this? Since Δ always means the ending value minus the beginning value, a negative result for Δv means that the final speed was *less than* the initial speed, so the car slowed down. This leads to a negative value for $\Delta v/\Delta t$. Since the speed decreased at the same rate for the last 3 s, this was uniform negative acceleration, i.e., deceleration. If we look at the vectors involved, the negative value of the acceleration means that the acceleration vector is now opposing the velocity vector, not helping it. So the two vectors point in the opposite directions, as in Figure 1.12.

Can you summarize what the car was doing during this entire experiment? . . . Here is one way of putting it: When the experiment began, the car was already going at 10 km/hr, and accelerated uniformly to 50 km/hr in 5 s at an average rate of 8.0 km/hr/s (8.0 km/hr in each second). The acceleration and velocity vectors pointed in the same direction. It then cruised at that speed for 2 s, so the acceleration was zero. Then it braked uniformly at the average rate of -15 km/hr/s for 3 s, ending with a speed of 5 km/hr. The acceleration and velocity vectors were pointing in the opposite directions during the slowing of the car.

Just as with distance and time, we can obtain a "picture" of the speed during this overall motion by drawing a graph of the motion, with the instantaneous speeds (neglect direction for now) on the vertical axis and the corresponding times on the horizontal axis. The result for this example is shown in Figure 1.13. We have also connected the data points together with straight lines. Again, the steepness of the lines has an important meaning. We can see that the graph starts out at 10 km/hr on the v-axis and climbs steadily upward to 50 km/hr at 5 s. During this time it had positive acceleration. Then the graph becomes "flat." As time increases, the speed does not change. There is no rise or fall of the line, so the acceleration is zero. Then the graph starts to fall. Speed is changing, but the change in values is *downward*, so the motion involves slowing, so the acceleration is negative. The steepness of the line and its upward or downward slopes appear to indicate the amount of positive or negative acceleration. When it is horizontal, the acceleration is zero.

As you learned earlier, we can obtain a quantitative measure of the "steepness" of any line on a graph by obtaining the "slope." Remember that the

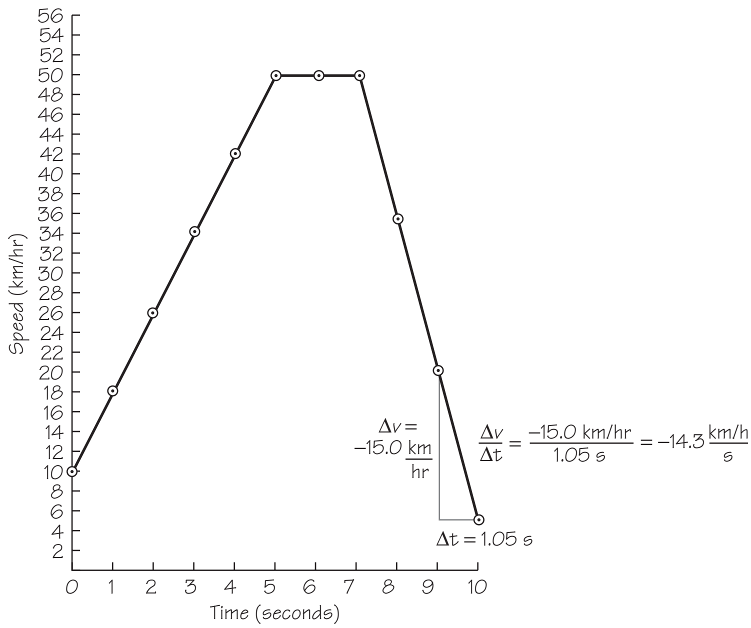


FIGURE 1.13 Speed vs. time graph.

slope is defined as the change in the y coordinate divided by the change in the x coordinate, slope $\equiv \Delta y/\Delta x$. In this case v is along the y-axis and t is along the x-axis, so the slopes of the line segments, $\Delta v/\Delta t$, are equal to the average acceleration, a_{av}, as we discussed before. In other words, *the slope of any straight-line part of a graph of instantaneous speed versus time gives a measure of the average acceleration of the object during that time interval.*

You can confirm the results in the last column of the table on page 33 by calculating the slopes of the lines in the graph.

1.7 FALLING FREELY

One of the most common occurrences of accelerated motion—yet one of the most difficult to study—is free fall, an object falling freely to the ground. An example is a ball dropped to the floor from a certain height. Just watching it fall, it is extremely difficult to see what is going on without modern equipment, since the ball drops so quickly. Happily today we do have equipment that helps us analyze the motion. The picture in Figure 1.14 is taken with the stroboscopic flash set at intervals of just 0.035 s and a camera with

We can obtain some simple equations *for the case of uniform acceleration*. If the average acceleration a_{av} is uniform, we can treat it as a mathematical constant and give it the symbol a. Thus,

$$\frac{\Delta v}{\Delta t} = a_{av} = a \quad \text{if } a_{av} \text{ is uniform.}$$

So,

$$\Delta v = a\,\Delta t$$

or

$$v = v_0 + a\,\Delta t.$$

In words, this result says that—for uniform acceleration—the instantaneous velocity (v) after a time interval of Δt is equal to the initial instantaneous velocity (v_0) plus the uniform acceleration (a) times the time interval (Δt).

Now, if the initial velocity was zero and if the clock was started at a reading of 0 s, this can be reduced even further to the simple formula

$$v = at.$$

But, as before, in order to use this simple formula the situation must satisfy all of the "ifs" in the discussed text (such as time starting at 0 s). Otherwise, when we don't know if the situation satisfies all of these "ifs," or when it satisfies only some of them, we must use the most general formula to define acceleration, which is

$$a_{av} = \frac{\Delta v}{\Delta t}.$$

an open shutter. This shows the position of the ball every 0.035 s against a metric ruler in the background.

How do you know that this ball is accelerating? Go back to the definition of acceleration in the previous section: a constantly increasing speed during equal time intervals. You can tell that the average speed is increasing because the distance traveled in each time interval is getting larger and larger. Free fall is an example of acceleration. In fact, as you will see, it is ideally an example of uniform acceleration. But to understand all of this, and without access to modern-day cameras and stroboscopic flashes, it took a person of the stature and genius of Galileo Galilei, who lived in Italy during the late sixteenth and early seventeenth centuries. What made Galileo different from many of his predecessors, especially those who followed Aristotle, and enabled his breakthrough, is his discovery that experimentation is the proper way to investigate nature, and that mathematics is the proper language for understanding and describing the laws of physics. "The book of nature is written in mathematical symbols," Galileo once said. Rather than qualitative arguments, Galileo relied upon the quantitative investiga-

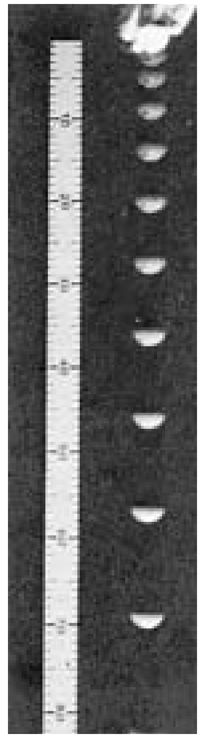


FIGURE 1.14 Stroboscopic photograph of a ball falling next to a vertical meter stick.

tion of physical events, just as physicists do today. We will follow his reasoning and discoveries, because he was the one who laid the foundations for the modern science of motion. In so doing, his view of nature, his way of thinking, his use of mathematics, and his reliance upon experimental tests set the style for modern physics in general. These aspects of his work are as important for understanding today's physics as are the actual results of his investigation.

1.8 *TWO NEW SCIENCES*

Galileo was old, sick, and nearly blind at the time he wrote *Two New Sciences*, which presented the new understanding of acceleration and free fall, and many other topics regarding motion. Yet, as in all his writings, his style is lively and delightful. He was also one of the very few authors of that time to write and publish in the vernacular, that is in Italian, rather than in the scholarly Latin. This indicated that he was writing as much for the educated Italian public as he was for a circle of academic specialists.

Probably influenced by his reading of Plato's dialogues, Galileo presented his ideas in *Two New Sciences* in the form of a dialogue, or conversation, among three fictional speakers. One of the speakers, named Simplicio, represented Aristotle's views. The proximity of his name to "simplicissimo," "the most simple one" in Italian, was surely no accident, although he was not made out to be a fool but a sophisticated Aristotelian philosopher. The other two fictional characters were Salviati, who represented Galileo himself, and Sagredo, a man of good will and open mind, eager to learn. Eventually, of course, Salviati leads Sagredo to Galileo's views and away from Simplicio's Aristotelian ideas.

The three friends first tackle the difficult problem of free fall. Aristotle's views on this subject still dominated at that time. According to Aristotle, each of the four elements has a natural place where it "belongs" and to which it will return on a straight line if removed from its natural place. Thus, a stone raised up into the air will, when released, drop straight down to the Earth. The heavier it is, the faster it will drop because it has more "earth" element in it, although air resistance will slow it down a little. Thus, Simplicio argued in Galileo's book, when a cannonball and a bird shot are dropped simultaneously from the same height, the cannonball will hit the ground much sooner than the bird shot.

This does sound very reasonable, and in fact different bodies falling from the same height may not reach the ground at exactly the same time. But the difference is not the huge difference predicted by Aristotle, but a minor difference which Galileo correctly attributed to the effect of air resistance on bodies of different size and weight. It is a further characteristic of Galileo's genius that he was able to recognize that the effects of air resistance and friction, though present in most real experiments, should be neglected so that the important feature of free fall—that in the absence of air resistance all objects fall with the same acceleration—is not overlooked. (We, too, neglected any friction and air resistance in the earlier disk experiment in order to observe the essential features of the motion.)

Aristotle regarded air resistance as such an important component of free fall that for him it had a major impact on the motion. He was right when we compare, for instance, a falling sheet of paper with a falling book. The book *does* reach the ground long before the paper does! But here again, it takes a special insight to realize that, while air resistance is a major factor for the falling paper, it is not so for the falling book. Hence the book and the paper are falling under two different circumstances, and the result is that one hits the ground a lot sooner than the other. Make the air resistance on the paper equivalent to that on

Aristotle: Rate of fall is proportional to weight divided by resistance.

FIGURE 1.15 A falling leaf.

the book by crumpling the paper into a tight ball and then try the experiment again. You will see a big difference from the previous case!

Galileo's conclusion that all falling objects fall with the same acceleration if air resistance is neglected depended on his being able to imagine how two objects would fall if there were no air resistance. His result seems simple today, when we know about vacuum pumps and the near vacuum of outer space, where there is no air resistance. But in Galileo's day a vacuum could not be achieved, and his conclusion was at first very difficult to accept.

A few years after Galileo's death, the invention of the vacuum pump allowed others to show that Galileo was indeed right! In one experiment, a feather and a heavy gold coin were dropped from the same height at the same time inside a container pumped almost empty of air. With the effect of air resistance eliminated, the different bodies fell at exactly the same rate and struck the bottom of the container at the same instant. Centuries later, when the Apollo astronauts landed on the Moon, they performed a famous experiment in which they dropped a feather and a hammer (a gold coin was not available . . . or deemed too expensive) simultaneously from the same height in the vacuum of space on the surface of the Moon. Just as Galileo would have predicted, the feather and the hammer hit the Moon's dusty surface at the exact same time!

1.9 FALLING OBJECTS

After establishing that under ideal conditions falling objects hit the ground at the same time when dropped from the same height, Galileo then had his alter ego, Salviati, hypothesize that a freely falling cannonball or any heavy object falls at *uniform* acceleration. But how exactly should one define uniform acceleration (after all, Galileo was the first one to put these ideas on paper)? Should one base the definition on the distance the object travels, or on the time it takes to travel? It was up to Galileo to decide, and he chose the definition that has been accepted ever since. Galileo declared in the book, *Two New Sciences*:

> A motion is said to be uniformly accelerated when, starting from rest, it acquires equal increments of speed Δv during equal time intervals Δt.

In other words, for uniform acceleration the ratio $\Delta v/\Delta t$ would be constant during any portion of the accelerated motion.

Galileo then set out to show that this definition held in the case of falling objects. But he knew that there was a practical problem. Suppose you drop a heavy ball from a given height to test whether the ratio $\Delta v/\Delta t$ for its motion really is constant for the entire path of the falling ball. To obtain the value of the ratio, you would have to measure the instantaneous speed of the ball and the corresponding elapsed time at different points along the path of fall. You would then obtain the values of Δv and Δt for pairs of points along the path and divide the results to see if they yield the same constant value.

In reality this experiment was impossible to perform. Even with modern instruments, such as those that enabled us to create the earlier photographs, it is difficult to measure the speed of a falling object. Furthermore, the time intervals involved are so short that Galileo could not have measured them accurately with the timing devices available to him. Even a ball dropped off a 10-story building takes less than 6 s to cover the entire distance to the ground. So a direct test of the ratio $\Delta v/\Delta t$ was impossible then, and it is not easy even today.

This did not stop Galileo. He turned to mathematics, the "language of nature," in order to obtain from his hypothesis that uniform acceleration governs the fall of objects some other relationship that could be checked by measurements with the equipment available to him. Using a little bit of geometry then—we would use algebra today (see the discussion in the *Student Guide*)—Galileo eliminated the change of speed Δv from the defining formula for acceleration. He did this by expressing the uniform acceleration a for an object starting from rest in a relationship involving the distance traveled Δd and the elapsed time Δt, both of which are quantities that are easier to measure than Δv:

$$\Delta d = \tfrac{1}{2}\, a(\Delta t)^2$$

or, regrouping terms,

$$a = \frac{2\Delta d}{\Delta t^2}.$$

This is the kind of relation Galileo was seeking. It relates the total distance traveled Δd and the total elapsed time Δt to the acceleration a, without involving any speed term.

Before finishing, though, we can simplify the symbols in either equation to make the equations easier to use. If you measure distance and time from the position and the instant that the motions starts, then $d_{\text{initial}} = 0$ and $t_{\text{initial}} = 0$. Thus, the intervals Δd and Δt have the values given by d_{final} and t_{final}. We can then write the first equation above more simply as

$$d_{\text{final}} = \tfrac{1}{2}\, a t_{\text{final}}{}^2.$$

Or, if we simply write d_{final} as d and t_{final} as t, we have

$$d = \tfrac{1}{2}\, a t^2.$$

This is the most well-known form of Galileo's famous result, but remember that it is a very specialized equation. It gives the total distance fallen as a function of the total time of free fall, but it does so only if the motion starts from rest ($v_{\text{initial}} = 0$), if the acceleration is uniform (a = constant), and if time and distance are measured from the start of the fall ($t_{\text{initial}} = 0$ and $d_{\text{initial}} = 0$).

The Meaning of Galileo's Result

Let's take a moment to look at what the equation $d = \frac{1}{2}at^2$ really means. It says in words that, for constant acceleration (in free fall or any other motion), the total distance traveled is equal to one-half the acceleration times the square of the total elapsed time. Since a is a constant, the equation can also be expressed as a proportion, using the proportionality sign $\propto$:

$$d \propto t^2.$$

In this case, $\frac{1}{2}a$ is the constant of proportionality. For example, if a uniformly accelerating car moves 10 m in the first second, it will move 40 m in the second second, 90 m in the third second, and so on. All objects accelerating with an acceleration equal to that of this car will have the same results. Or, expressed differently, the ratio of d to t^2 will have the same value

$$\frac{d}{t^2} = \text{constant} = \tfrac{1}{2}a.$$

Conversely, any motion for which this ratio of d and t^2 is constant for different distances and their corresponding times is a case of *uniform acceleration*. Galileo used this last statement to test if the acceleration of free fall is indeed uniform.

Galileo Turns to an Indirect Test

Galileo encountered another practical problem in examining the acceleration of free fall: how to measure the very short time intervals as an object fell the different distances? (See the insert "Dropping a ball from the Leaning Tower of Pisa.") A direct test of the hypothesis was still not possible. Ingeniously, Galileo turned to a clever indirect test. He decided to test an object that was falling under the influence of gravity but not falling freely. He proposed a new hypothesis:

> If a freely falling body has uniform acceleration, then a perfectly round ball rolling down a perfectly smooth incline will also have a constant, though smaller, acceleration.

Galileo's Salviati described just such an experimental test in *Two New Sciences*. Others who repeated this experiment have obtained results very similar to those he described. (You might perform this experiment yourself, or a similar one, in the laboratory.)

DROPPING A BALL FROM THE LEANING TOWER OF PISA

Although there is no certain evidence that Galileo actually dropped a ball from the Leaning Tower of Pisa, we can use his equation to find out how long it would take for a ball to reach the ground.

The Leaning Tower is about 58.4 m in length, but the actual distance to the ground is less, because of the lean. Today the amount of lean of the top with respect to the base is about 5.20 m (this may change soon, when efforts are made to stabilize the tower). Using the Pythagorean theorem, today the distance straight down from the top can be calculated to be 58.2 m. The acceleration of the ball is the acceleration of gravity, which has the symbol g. The acceleration of gravity is $g = 9.8\ \mathrm{m/s^2}$.

Using Galileo's equation for a ball dropped from rest, $d = \frac{1}{2}at^2$, the time for the ball to fall this distance, neglecting air resistance, may be found as follows:

$$d = \tfrac{1}{2}at^2,$$

$$t = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2(58.2\ \mathrm{m})}{9.8\ \mathrm{m/s^2}}}$$

$$= \sqrt{11.9\ \mathrm{s^2}}$$

$$= 3.5\ \mathrm{s}.$$

How fast would the ball be moving the instant before it hits the ground (again, neglecting air resistance)?

$$v = at = gt = (9.8\ \mathrm{m/s^2})(3.5\ \mathrm{s})$$

$$= 34.3\ \mathrm{m/s}.$$

This is very fast, indeed!

First, keeping a constant angle on an inclined plane, Galileo allowed a smooth round ball to roll different distances down the inclined plane from rest. He measured the elapsed time in each case using a water clock, a device that measures elapsed time by the amount of water that flows at a steady rate from a vessel. If d_1, d_2, and d_3 represent distances reached by the ball, measured from the starting point on the incline, and t_1, t_2, and t_3 represent the corresponding times taken to roll down these distances, then he could see whether the accelerations had the same value in each of the cases by dividing each d by the corresponding t^2. If these ratios all had the same value, then, as just discussed, this would be a case of uniform accel-

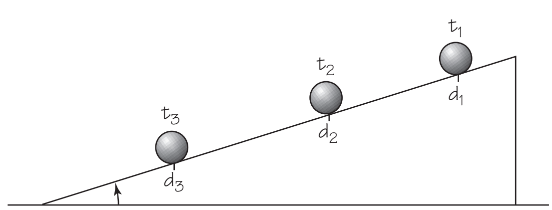


FIGURE 1.16 Ball rolling down inclined plane.

FIGURE 1.17 This 1841 painting by Guiseppi Bezzuoli attempts to reconstruct an experiment Galileo is alleged to have made during his time as lecturer at Pisa. Off to the left and right are men of ill will: the blasé Prince Giovanni de Medici (Galileo had shown a dredging machine invented by the prince to be unusable) along with Galileo's scientific opponents. These were leading men of the universities; they are shown here bending over a book of Aristotle, in which it is written in black and white that bodies of unequal weight fall with different speeds. Galileo, the tallest figure left of center in the picture, is surrounded by a group of students and followers.

eration. (In another account, Galileo kept time by singing a song and beating time with his fingers while rolling cannonballs on the inclined plane.

Whichever method he used for measuring time, Galileo obtained precisely the predicted result for the relationship between the distance traveled and the elapsed time when the angle of the incline is unchanged. He concluded from this that since the value of d/t^2 was constant for a given angle of incline, the acceleration for each incline is indeed uniform.

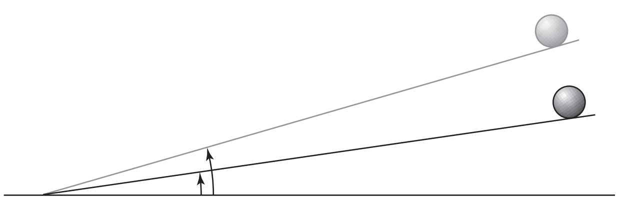
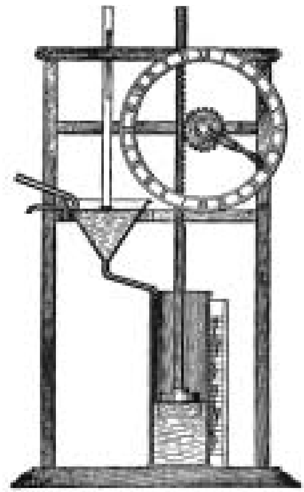


FIGURE 1.18 Ball rolling down inclined planes.

FIGURE 1.19 An early water clock.

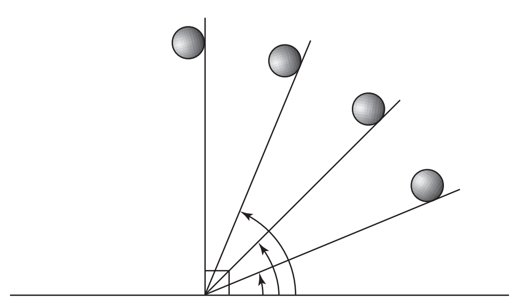


In a second series of experiments, Galileo examined what happens when the angle of inclination of the plane is changed. As the angle of incline increased, Galileo reported that the ratio d/t^2 also increased.

After obtaining his results for small inclines, Galileo then tried a "thought experiment," that is, an experiment that he performed only logically in his mind, since it could not be carried out in reality. In this thought experiment, Galileo extrapolated his results for small angles of incline to steeper and steeper angles where the ball moves too fast for accurate measurements of t, and finally to the angle of inclination of 90°, when the ball would be moving straight down as a freely falling object. Since this was just the extreme case of motion on the incline (an "incline" that is actually vertical), he reasoned that d/t^2 would still be constant even in that extreme case.

In short, by the experiment on the inclined plane Galileo had found that a constant value of d/t^2 would be characteristic of uniform acceleration

FIGURE 1.20 Spheres rolling down planes of increasingly steep inclination. For each angle, the acceleration has its own constant value. At 90 degrees, the inclined plane situation looks almost like free fall. Galileo assumed that the difference between free fall and "rolling fall" is not important (in most real situations, the ball would slide, not roll, down the steep inclines).



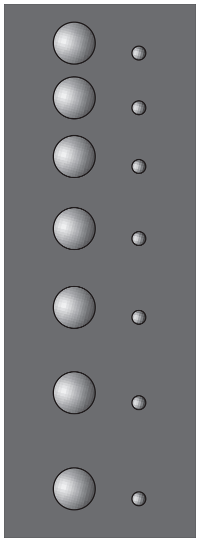


FIGURE 1.21 Two falling balls (unequal weights) in vacuum.

starting from rest. By extrapolation that experimental situation which he could measure, he concluded at last that free fall, which he could not examine directly, was an example of uniformly accelerated motion. Therefore, neglecting air resistance, Galileo concluded:

> all freely falling objects, whether heavy or light, will cover the same distance in the same amount of time, moving with the same constant acceleration. They will hit the ground together if they are dropped together from the same height.

This conclusion has been tested many times with modern apparatus (e.g., Figure 1.21), and it is sometimes called "Galileo's law (or rule) of free fall."

1.10 THE CONSEQUENCES

The results of Galileo's work on speed, acceleration, and falling bodies were most important to the development of physics, and they are now a fundamental part of today's physics. But they could scarcely have brought about a revolution in science by themselves. No sensible scholar in the seventeenth century would have given up a belief in Aristotle's cosmology only because some of its predictions had been disproved. Moreover, Galileo did not explain *why* objects fall and move as they do; he was providing only a *description* of these motions. In technical terms, Galileo provided what is known as the *kinematics* (description) of motion, not the *dynamics* (causes) of motion. Together, kinematics and dynamics form the field of *mechanics* in physics.

Still, even without going further, Galileo's understanding of free-fall motion helped to prepare the way for a new kind of physics, and indeed for a completely new cosmology, by planting the seeds of doubt about the basic assumptions of Aristotle's science and providing an alternative description.

The most disputed scientific problem during Galileo's lifetime was not in mechanics but in astronomy. A central question in astronomy was whether the Earth or the Sun was at the center of the Universe. Galileo supported the view that the Earth and other planets revolved around the Sun, a view entirely contrary to Aristotle's cosmology in which the Earth was at the center and the Sun and planets revolved around it. But the new astronomy required a new physical theory of why and how the Earth itself moved. Galileo's work on free fall and other motions turned out to be just what was needed to begin constructing such a theory. His work did not have its full effect, however, until it had been combined with studies of the causes of motion (forces) by the English scientist Isaac Newton. As Galileo had done earlier, Newton combined reason, mathematics, and experiment into the extremely capable tool of research in physics that we have today. Galileo himself foresaw that this was going to happen. In his book *Two New Sciences*, after summarizing the new method, Galileo had his alter ego Salviati, proclaim:

> We may say the door is now opened, for the first time, to a new method fraught with numerous and wonderful results which in future years will command the attention of other minds.

SOME NEW IDEAS AND CONCEPTS

acceleration
average speed
dynamics
Galileo's law of free fall
instantaneous speed
kinematics
mechanics
scalar
speed
thought experiment
uniform
vector
velocity

SOME FORMULAS FOR MOTION AND THEIR MEANING

This list is provided only as a reference. It is not intended that you merely memorize these formulas; but you should know what they mean.

$\Delta d = d_{\text{final}} - d_{\text{initial}}$	The distance traveled is the final position minus the initial position.
$\Delta t = \mathrm{t}_{\text{final}} - \mathrm{t}_{\text{initial}}$	The time interval is the finishing time minus the starting time.
$v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$	The average speed is the ratio of the distance traveled per time interval.
$\text{slope} = \dfrac{\Delta y}{\Delta x}$	The slope of a line on a graph between two points is defined as the ratio of the change in the y coordinate to the change in the x coordinate between the two points.
$v = \lim_{\Delta t \to 0} \dfrac{\Delta d}{\Delta t}$	The instantaneous speed is the limit, as the time interval approaches zero, of the ratio of the distance traveled per time interval.
$a_{\text{av}} = \dfrac{\Delta v}{\Delta t}$	The average acceleration is the ratio of the change in speed and the time interval.
$\mathbf{a}_{\text{av}} = \dfrac{\Delta \mathbf{v}}{\Delta t}$	The average vector acceleration is the ratio of the change in velocity per time interval.
$v = at$	If an object is accelerating uniformly from zero initial speed starting at time zero, the speed at time t is the acceleration times the time.
$\Delta d = \frac{1}{2} a(\Delta t)^2$	For uniform acceleration only and an object starting from rest, the distance traveled is one-half the acceleration times the square of the time interval.
$d = \frac{1}{2} at^2$	For objects accelerating uniformly from rest and from an initial distance and time of zero, the distance traveled during time t is one-half the acceleration times the square of the elapsed time.

FURTHER READING

G. Holton and S.G. Brush, *Physics, The Human Adventure* (Piscataway, NJ: Rutgers University Press, 2001), Chapters 6 and 7.

Web site

See the course Web site at: http://www.springer-ny.com/up.

STUDY GUIDE QUESTIONS*

1. Summarize in writing the main points of each section.
2. Compare the sections on the basic concepts of motion in this chapter with your laboratory work.
3. Write down any concepts or formulas or paragraphs that you do not understand and ask your instructor for help.
4. Look at each of the tables in this chapter. Explain each of these tables to a friend, including the meaning of each row and column and how the numbers were obtained.

Sections 1.1–1.7

1. What is meant by the Scientific Revolution?
2. Who was Galileo and what did he do?
3. Why didn't Galileo continue to write about astronomy during his lifetime?
4. Define in your own words the following terms: speed, velocity, average speed, average acceleration, uniform acceleration, and the symbol Δd.
5. Give an example of each of the concepts in Question 4.
6. What is free fall? What is "free" about free fall? Why is it so difficult to determine if free fall is an example of uniform acceleration?
7. How did Galileo's approach to the problem of motion differ from that of Aristotle and of the Aristotelians in his own day?
8. Which of the following properties do you believe might affect the observed rate of fall of an object: weight, size, mass, color, shape, or density.
9. A ball is rolling on the floor with a constant speed of 130 cm/s. It starts rolling at time 0.0 s and at distance 0.0 cm.
   (a) Calculate where it would be at every one-tenth of a second up to 1.0 s. Construct a table of distance and time to show your results.
   (b) Add additional columns to your table showing the distance covered in each time interval, and the average speed in each time interval.

* These questions are intended as an aid to study. Your instructor may ask you to work on these in groups, or individually, or as a class.

(c) Draw a graph of total distance traveled against the elapsed time.
(d) From the table, predict what the slope of the line should be.
(e) Find the slope of the line and compare the result with your prediction. Explain your result.

10. A car is increasing its speed uniformly at the rate of 8.0 km/hr each second. It starts out at 0 km/hr at time 0 s.
 (a) Calculate what its instantaneous speed would be at every second up to 10 s. Construct a table of speed and time to show your results.
 (b) Add additional columns to your table showing the increase of speed in each time interval, and the average rate of increase in each time interval.
 (c) Draw a graph of instantaneous speed against the elapsed time.
 (d) From the table, predict what the slope of the line should be.
 (e) Find the slope of the line and compare with your prediction. Explain your result.

Sections 1.8–1.10

1. What was Aristotle's theory about falling objects?
2. What did Galileo claim would happen if he dropped two different heavy objects simultaneously from the same height?
3. Why was Galileo slightly wrong? Why didn't this force him to change his mind?
4. If you drop a sheet of paper and a book simultaneously from the same height, they don't reach the ground together. Why not? Could one change the experiment so that they do reach the ground together?
5. What is the definition that Galileo chose for uniform acceleration?
6. What was Galileo's hypothesis about free fall?
7. Why couldn't Galileo test his hypothesis directly?
8. What procedure did he use to test it indirectly?
9. What is Galileo's law of free fall and how did he arrive at this conclusion?
10. What are some of the consequences of Galileo's work?
11. What are some important elements that go into scientific research? How are they represented in Galileo's work?

DISCOVERY QUESTIONS*

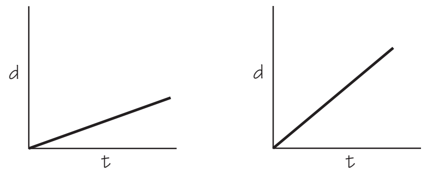
†1. Why do you think motion is so important to understanding nature?

†2. During the course of one day notice the different types of motion that you encounter and write down your observations in a notebook. Try to classify these motions according to the type of motion involved: uniform speed, uniform velocity, accelerated motion, or combinations of these.

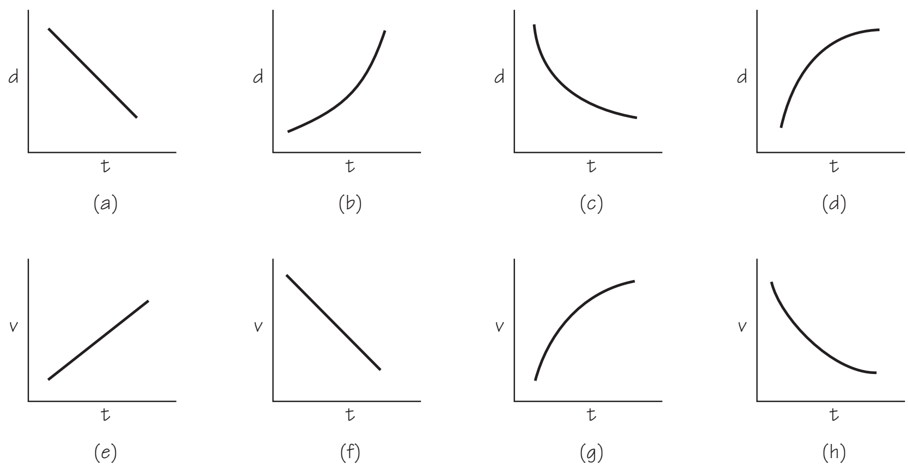
* Some of these questions go beyond the text itself.

† These may be performed before or during the reading of this chapter.

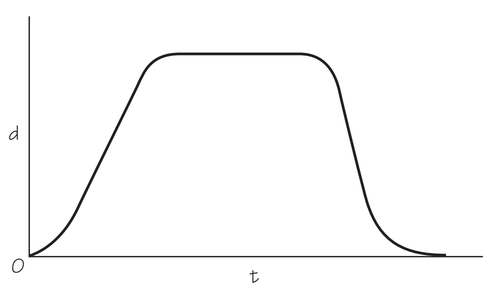
3. Which of the two graphs below, for two different objects, has the greater slope.



4. Explain the difference between average speed and instantaneous speed.
5. Shown below are graphs representing different motions of a disk on a tabletop. Explain what is happening in each case, and how you arrived at your answer. The axes and the slope of the lines will help to answer this question.



6. Make up a story to go along with the motion that is represented by the distance–time graph below.



7. Galileo wrote: “A motion is said to be uniformly accelerated when, starting from rest, it acquires during equal time intervals, equal increments of speed.” Using modern symbols, this says: uniform acceleration involves equal values

of Δv in equal time intervals Δt. Which of the following are other ways of expressing the same idea?
(a) Δv is proportional to Δt;
(b) $\Delta v/\Delta t$ = constant;
(c) the speed–time graph is a straight line;
(d) v is proportional to t.

8. Suppose Galileo performed his famous inclined plane experiment on the Moon, where the acceleration owing to gravity is less. Would he have obtained the same law of free fall?

Quantitative

1. Time yourself in going from one class to another, then the next time pace off the distance to obtain a rough estimate of the total distance. Find your average speed from these results.
2. A certain person who walks for exercise usually walks 2 mi (3.23 km) in about 28 min. What is his/her average speed in kilometers per second and miles per hour?
3. Amtrak runs a high-speed train, the Acela Express, between New York and Washington. The train can cover the distance of 194 mi in 2 hr, 43 min. What is its average speed?

Amtrak Acela Express.

4. (a) A world record for the mile, set in 1999, was 3 min, 43 s. What was the average speed of the runner in miles per hour?
   (b) The cheetah is the fastest animal on Earth. It can run at a sustained average speed of 80 mi/hr. What would be its time for 1 mi?

5. The winner of the last stage of the 1999 Tour de France, a French cyclist, covered the distance of 143.5 km from Arpajon to Paris in 3 hr 37 min 39 s. What was his average speed in kilometers per hour?

6. What is the average acceleration of an airplane that goes from 0 to 100 km/hr in 5 s during take off? What is the average acceleration during landing, going from 200 km/hr in 12 s?
7. (a) A rollerblader starts out at the top of a hill with a 200 m slope. She rolls, without skating, to the bottom of the hill in 15 s. What was her acceleration?

(b) What was her speed at the bottom of the hill?
(c) The rollerblader skates halfway back up the hill, then rolls down to the bottom. What was her acceleration on the way down? What was her speed at the bottom?

8. (a) Light waves travel through a vacuum in a straight line at a speed of about 3×10^8 m/s. How far is a "light year"?
(b) The nearest star, Alpha Centauri, is about 4×10^{16} m distant from Earth. This star does not have any planets, but if it did and intelligent beings lived on them, how soon, at the earliest, could we expect to receive a reply after sending them a light signal strong enough to be received there?
9. The speed of sound in air is about 330 m/s (0.33 km/s). Suppose lightning strikes 1 km away. How long does it take before you see the flash? (The speed of light is given in 8a.) How long before you hear the thunder? On the basis of your results, can you set up a rule for determining how far away a lightning flash occurred if you count the seconds between the sight of the flash and the arrival of the sound of thunder?
10. Ask a friend who has access to a car to drive with you on a quiet stretch of straight road with little or no traffic or pedestrians. Take data on the car's speed. After attaining a speed of about 30 mi/hr, ask that the car slow down and record the speedometer readings in 5-s intervals.
(a) After returning, create a table of speed and corresponding time increments.
(b) Add the following columns to your table: the change in speed Δv, the time interval Δt, and the average acceleration in each time interval $\Delta v/\Delta t$.
(c) Create a graph of speed and time for the motion of the car. Find the average acceleration of the car from the graph. How does it compare with the results from your table?