CHAPTER 5

Conserving Matter and Motion

5.1 CONSERVATION OF MASS

Newton's success in mechanics altered profoundly the way in which scientists viewed the Universe. The motions of the Sun and planets could now be considered as purely mechanical, that is, governed by the laws of mechanics, much like a machine. As for any machine, whether a clock or the solar system, the motions of the parts were completely determined once the system had been put together.

This model of the solar system is called the *Newtonian world machine*. As is true of any model, certain things are left out. The mathematical equations that govern the motions of the model cover only the main properties of the real solar system. The masses, positions, and velocities of the parts

FIGURE 5.1 Newtonian physics inspired a mechanistic view of the universe as a self-contained "clock" designed by God to run on its own according to discernible principles and without any further need for Divine intervention (except, Newton thought, for occasional "fine tuning").

of the system, and the gravitational forces among them, are well described. But the Newtonian model neglects the internal structure and chemical composition of the planets, as well as heat, light, and electric and magnetic forces. Nevertheless, it serves splendidly to deal with observed motions in mechanics, and to this day is in constant use, in physics, engineering, sports, etc. Moreover, Newton's approach to science and many of his concepts became useful later in the study of those aspects he had to leave aside.

The idea of a world machine does not trace back only to Newton's work. In his *Principia Philosophiae* (1644), René Descartes, the most influential French philosopher of the seventeenth century, had written:

> I do not recognize any difference between the machines that artisans make and the different bodies that nature alone composes, unless it be that the effects of the machines depend only upon the adjustment of certain tubes or springs, or other instruments, that, having necessarily some proportion with the hands of those who make them, are always so large that their shapes and motions can be seen, while the tubes and springs that cause the effects of natural bodies are ordinarily too small to be perceived by our senses.

Robert Boyle (1627–1691), a British scientist, is known particularly for his studies of the properties of air. Boyle, a pious man, expressed the "mechanistic" viewpoint even in his religious writings. He argued that a God who could design a universe that ran by itself, as an ideal machine would, was more wonderful than a God who simply created several different kinds of matter and gave each a natural tendency to behave as it does. Boyle also thought it was insulting to God to believe that the world machine would be so badly designed as to require any further divine adjustment once it had been created. He suggested that an engineer's skill in designing "an elaborate engine" is more deserving of praise if the engine never needs supervision or repair. Therefore, if the "engine" of the Universe is to keep running unattended, the amounts of matter and motion in the Universe must remain constant over time. Today we would say that they must be *conserved*.

The idea that despite ever-present, obvious change all around us the total amount of material in the Universe does not change is really very old. It may be found, for instance, among the ancient atomists (see Prologue).

FIGURE 5.2 *The Ancient of Days* by William Blake (1757–1827), an English poet and artist who had little sympathy with the Newtonian style of "Natural Philosophy."

And just 24 years before Newton's birth, the English philosopher Francis Bacon included the following among his basic principles of modern science in *Novum Organum* 1620):

> There is nothing more true in nature than the twin propositions that "nothing is produced from nothing" and "nothing is reduced to nothing" . . . the sum total of matter remains unchanged, without increase or diminution.

This view agrees with everyday observation to some extent. While the form in which matter exists may change, in much of our ordinary experience matter appears somehow indestructible. For example, you may see a large boulder crushed to pebbles and not feel that the amount of matter in the Universe has diminished or increased. But what if an object is burned to ashes or dissolved in acid? Does the amount of matter remain unchanged even in such chemical reactions? What of large-scale changes such as the forming of rain clouds or seasonal variations?

In order to test whether the total quantity of matter actually remains constant, you must know how to measure that quantity. Clearly, it cannot be measured simply by its volume. For example, you might put water in a container, mark the water level, and then freeze the water. If you try this, you will find that the volume of the ice is greater than the volume of the water you started with. This is true even if you carefully seal the container

FIGURE 5.3 In some open-air chemical reactions, the mass of objects seems to decrease, while in others it seems to increase

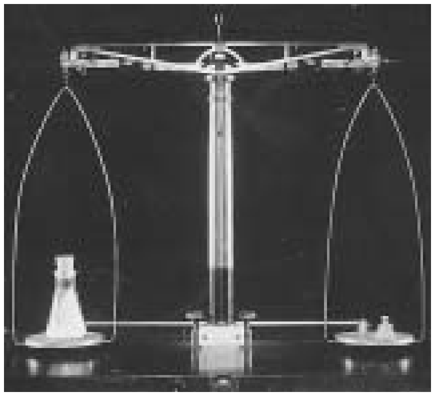


FIGURE 5.4 Conservation of mass was first demonstrated in experiments on chemical reactions in closed flasks.

so that no water can possibly come in from the outside. Similarly, suppose you compress some gas in a closed container. The volume of the gas decreases even though no gas escapes from the container.

Following Newton, we regard the *mass* of an object as the proper measure of the amount of matter it contains. In all the examples in previous chapters, we assumed that the mass of a given object does not change. However, a burnt match has a smaller mass than an unburnt one; an iron nail increases in mass as it rusts. Scientists had long assumed that something escapes from the match into the atmosphere and that something is added from the surroundings to the iron of the nail. Therefore, nothing is really "lost" or "created" in these changes. Not until the end of the eighteenth century was sound experimental evidence for this assumption provided. The French chemist Antoine Lavoisier produced this evidence.

Lavoisier (1743–1794), who is often called the "father of modern chemistry," closely examined chemical reactions that he caused to occur in *closed* flasks (a "closed system"). He carefully weighed the flasks and their contents before and after each reaction. For example, he burned iron in a closed flask. He found that the mass of the iron oxide produced equaled the sum of the masses of the iron and oxygen used in the reaction. With experimental evidence like this at hand, he could announce with confidence in *Traité Elémentaire de Chimie* (1789):

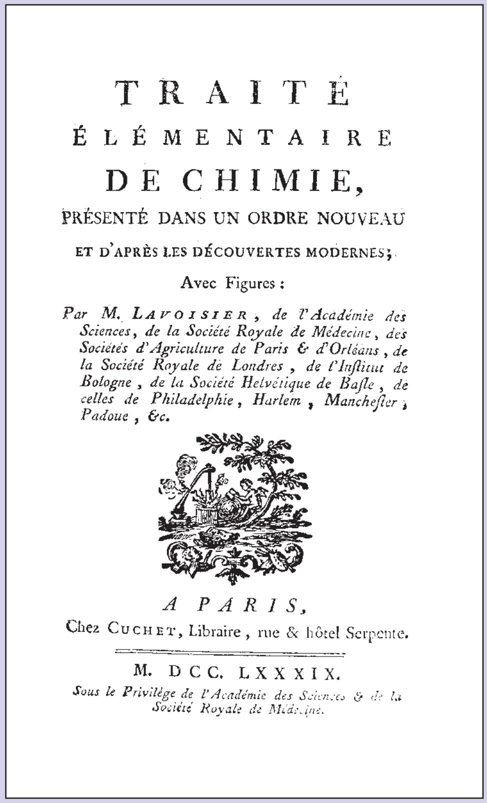
> We may lay it down as an incontestable axiom that in all the operations of art and nature, nothing is created; an equal quantity of matter exists both before and after the experiment . . . and nothing takes place beyond changes and modifications in the combinations of these elements. Upon this principle, the whole art of performing chemical experiments depends.

THE FATHER OF MODERN CHEMISTRY

Antoine Laurent Lavoisier showed the decisive importance of quantitative measurements, confirmed the principle of conservation of mass in chemical reactions, and helped develop the present system of nomenclature for the chemical elements. He also showed that organic processes such as digestion and respiration are similar to burning.

To earn money for his scientific research, Lavoisier invested in a private company which collected taxes for the French government. Because the tax collectors were allowed to keep any extra tax which they could collect from the public they became one of the most hated groups in France. Lavoisier was not directly engaged in tax collecting, but he had married the daughter of an important executive of the company, and his association with the company was one of the reasons why Lavoisier was guillotined during the French Revolution.

Also shown in the elegant portrait by J.L. David is Madame Lavoisier. She assisted her husband by taking data, translating scientific works from English into French, and making illustrations. About 10 years after her husband's execution, she married another scientist, Count Rumford, who is remembered for his experiments which cast doubt on the caloric theory of heat.



TRAITÉ
ÉLÉMENTAIRE
DE CHIMIE,
PRÉSENTÉ DANS UN ORDRE NOUVEAU
ET D'APRÈS LES DÉCOUVERTES MODERNES;
Avec Figures :

Par M. Lavoisier, de l'Académie des Sciences, de la Société Royale de Médecine, des Sociétés d'Agriculture de Paris & d'Orléans, de la Société Royale de Londres, de l'Institut de Bologne, de la Société Helvétique de Basle, de celles de Philadelphie, Harlem, Manchester, Padoue, &c.

A PARIS,
Chez Cuchet, Libraire, rue & hôtel Serpente.

M. DCC. LXXXIX.

Sous le Privilége de l'Académie des Sciences & de la Société Royale de Médecine.

FIGURE 5.6 Title page from Lavoisier's *Traite*.

FIGURE 5.5 The Lavoisiers.

Lavoisier knew that if he put some material in a well-sealed bottle and measured its mass, he could return at any later time and find the same mass. It would not matter what had happened to the material inside the bottle. It might change from solid to liquid or liquid to gas, change color or consistency, or even undergo violent chemical reactions. At least one thing would remain unchanged: the *total* mass of all the different materials in the bottle.

In the years after Lavoisier's pioneering work, a vast number of similar experiments were performed with ever-increasing accuracy. The result was always the same. As far as scientists now can measure with sensitive balances (having a precision of better than 0.000001%), mass is *conserved*, that is, remains constant, in chemical reactions.

To sum up, despite changes in location, shape, chemical composition, and so forth, *the mass of any closed system remains constant*. This is the statement of the *law of conservation of mass*. This law is basic to both physics and chemistry.

5.2 COLLISIONS

Looking at moving things in the world around us easily leads to the conclusion that everything set in motion eventually stops. Every actual machine, left to itself, eventually runs down. It appears that the amount of motion in the Universe must be decreasing. This suggests that the Universe, too, must be running down, though, as noted earlier, many philosophers of the seventeenth century could not accept such an idea. Some definition of "motion" was needed that would permit one to make the statement that "the quantity of motion in the Universe is constant."

Is there a constant "quantity of motion" that keeps the world machine going? To suggest an answer to this question, you can do some simple laboratory experiments (Figure 5.7). Use a pair of carts with equal mass and nearly frictionless wheels; even better are two dry-ice disks or two air-track gliders. In a first experiment, a lump of putty is attached so that the carts will stick together when they collide. The carts are each given a push so that they approach each other with equal speeds and collide head-on. As you will see when you do the experiment, both carts stop in the collision; their motion ceases. But is there anything related to their motions that does not change?

The answer is yes. If you add the velocity $\mathbf{v}_A$ of one cart to the velocity $\mathbf{v}_B$ of the other cart, you find that the *vector sum* does not change. The vector sum of the velocities of these oppositely moving equally massive carts is zero *before* the collision. It is also zero for the carts at rest *after* the collision.

Does this finding hold for all collisions? In other words, is there a "law of conservation of velocity"? The example above was a very special circumstance. Carts with equal masses approach each other with equal speeds. But suppose the mass of one of the carts is twice the mass of the other cart. We let the carts approach each other with equal speeds and collide, as before. This time the carts do *not* come to rest. There is some motion remaining. Both objects move together in the direction of the initial velocity of the more massive object. So the vector sum of the velocities is not conserved in all collisions. (See Figure 5.7.)

Another example of a collision will confirm this conclusion. This time let the first cart have twice the mass of the second, but only half the velocity. When the carts collide head-on and stick together, they stop. The vector sum of the velocities is equal to zero *after* the collision. But it was not equal to zero *before* the collision. Again, there is no conservation of velocity; the total "quantity of motion" is not always the same before and after a collision.

The problem was solved by Newton. He saw that the mass played a role in such collisions. He redefined the "quantity of motion" of a body as the product of its mass and its velocity, $m\mathbf{v}$. This being a vector, it includes the idea of the *direction* of motion as well as the speed. For example, in all

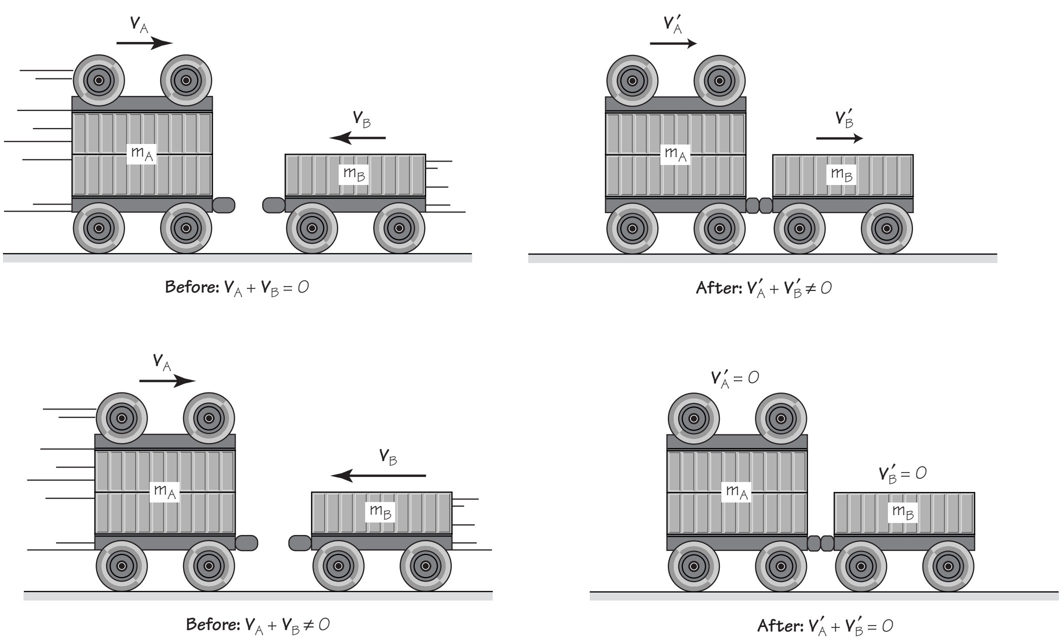


FIGURE 5.7 Collision of two carts (see text).

three collisions we have mentioned above, the motion of the carts before and after collision is described by the equation

$$\underbrace{m_A\mathbf{v}_A + m_B\mathbf{v}_B}_{\substack{\text{before}\\ \text{collision}}} = \underbrace{m_A\mathbf{v}'_A + m_B\mathbf{v}'_B}_{\substack{\text{after}\\ \text{collision}}}$$

Here m_A and m_B (which remain constant) represent the respective masses of the two carts, $\mathbf{v}_A$ and $\mathbf{v}_B$ represent their velocities before the collision, and $\mathbf{v}'_A$ and $\mathbf{v}'_B$ represent their velocities after the collision. Earlier, we represented initial and final velocities by $\mathbf{v}_i$ and $\mathbf{v}_f$. Here they are represented by $\mathbf{v}$ and $\mathbf{v}'$ because we now need to add subscripts, such as A and B.

In words, the above equation states:

> The *vector sum of the quantities mass* × *velocity before the collision is equal to the vector sum of the quantities mass* × *velocity after the collision. The vector sum of these quantities is constant*, *or conserved*, *in all these collisions*.

The above equation is very important and useful, leading directly to a powerful law, and of course is useful in allowing us to predict, at least qualitatively, the motions after collisions of the two colliding carts in the above examples.

5.3 CONSERVATION OF MOMENTUM

The product of mass and velocity often plays an important role in mechanics. It therefore has been given a special name. Instead of being called "quantity of motion," as in Newton's time, it is now called *momentum*. The total momentum of a system of objects (e.g., the two carts) is the vector sum of the momenta of all objects in the system. Consider each of the collisions examined. The momentum of the system as a whole, that is, the vector sum of the individual parts, is the same before and after collision. Thus, the results of the experiments can be summarized briefly: The momentum of the system is conserved.

This rule (or law, or principle) follows from observations of special cases, such as that of collisions between two carts that stuck together after colliding. But in fact, this *law of conservation of momentum* (often abbreviated LCM) turns out to be a completely general, universal law. The momentum of *any* system is conserved *if one condition is met:* that no net force is acting on the system—or, to put it in other words, that the system of objects can be considered closed to any effect from outside the system.

To see just what this condition means, let's examine the forces acting on one of the carts in the earlier experiment. Each cart, on a level track, experiences three main forces. There is, of course, a downward pull $\mathbf{F}_{\text{grav}}$ exerted by the Earth, and an equally large upward push $\mathbf{F}_{\text{table}}$ exerted by the table. (See Figure 5.8.) During the collision, there is also on each a push $\mathbf{F}_{\text{from other cart}}$ exerted by the other cart. The first two forces evidently cancel, since the cart is not accelerating up or down while on the tabletop. Thus, the net force on each cart is just the force exerted on it by the other cart as they collide. (We assume that frictional forces exerted by the table and the air are small enough to neglect. That was the reason for using dry-ice disks, air-track gliders, or carts with "frictionless" wheels. This assumption makes it easier to discuss the law of conservation of momentum. Later, you will see that the law holds whether friction exists or not.)

The two carts form a *system* of bodies, each cart being a part of the system. The force exerted by one cart on the other cart is a force exerted by one part of the system on another part. It is *not* a force on the system as a whole. The outside forces acting on the carts (by the Earth and by the table) exactly cancel. Thus, there is no *net* outside force. The system is "isolated." If this condition is met, the total momentum of all parts making up the system stays constant, it is "conserved." This is the *law of conservation of momentum* for systems of bodies that are moving with linear velocity **v**.

The Universality of Momentum Conservation

So far, you have considered only cases in which two bodies collide directly and stick together. The remarkable thing about the law of conservation of momentum is how universally it applies. For example:

1. It holds true no matter what *kind* of forces the bodies exert on each other. They may be gravitational forces, electric or magnetic forces,

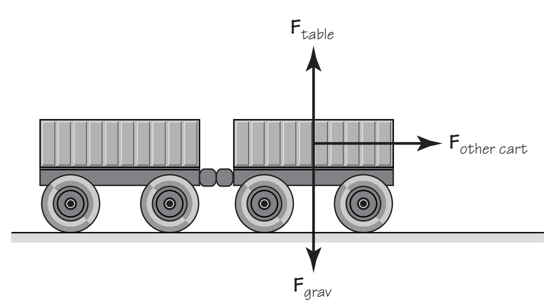


FIGURE 5.8 Forces on one of the carts during collision.

tension in strings, compression in springs, attraction or repulsion. The sum of the (mass × velocity) before is equal to the sum of the (mass × velocity) of all parts after any interaction.

2. The LCM also holds true even when there are friction forces present. If a moving object is slowed or stopped by frictional forces, for example, a book sliding to a stop on a tabletop, then the Earth, to which the table is attached, will take up the initial momentum of the book. In general, the object producing friction becomes part of the system of bodies to which the LCM applies.
3. It does not matter whether the bodies stick together or scrape against each other or bounce apart. They do not even have to touch. When two strong magnets repel or when a positively charged alpha particle is repelled by a nucleus (which is also positive), conservation of momentum still holds in each of those systems.
4. The law is not restricted to systems of only two objects; there can be any number of objects in the system. In those cases, the basic conservation equation is made more general simply by adding a term for each object to both sides of the equation.
5. The size of the system is not important. The law applies to a galaxy as well as to atoms.
6. The angle of the collision does not matter. All of the examples so far have involved collisions between two bodies moving along the same straight line. They were "one-dimensional collisions." If two bodies make a glancing collision rather than a head-on collision, each will move off at an angle to the line of approach. The law of conservation of momentum applies to such "two-dimensional collisions" also. (Remember that momentum is a vector quantity.) The law of conservation of momentum also applies in *three* dimensions. The vector sum of the momenta is still the same before and after the collision.

> In general symbols, for n objects, this law may be written:
> $$\sum_{i=1}^{n}(m_i\mathbf{v}_i)_{\text{before}} = \sum_{i=1}^{n}(m_i\mathbf{v}_i)_{\text{after}},$$
> where $\sum_i$ represents the sum of the quantities in parentheses.

In the *Student Guide* for this chapter you will find a worked-out example of a collision between a spaceship and a meteorite in outer space that will help you become familiar with the law of conservation of momentum. On p. 225, the sidebar, "A Collision in Two Dimensions," shows an analysis of a two-dimensional collision. There are also short VHS or DVD videos of colliding bodies and exploding objects. These include collisions and explosions in two and three dimensions. The more of them you analyze, the more convinced you will be that the law of conservation of momentum applies to *any* isolated system.

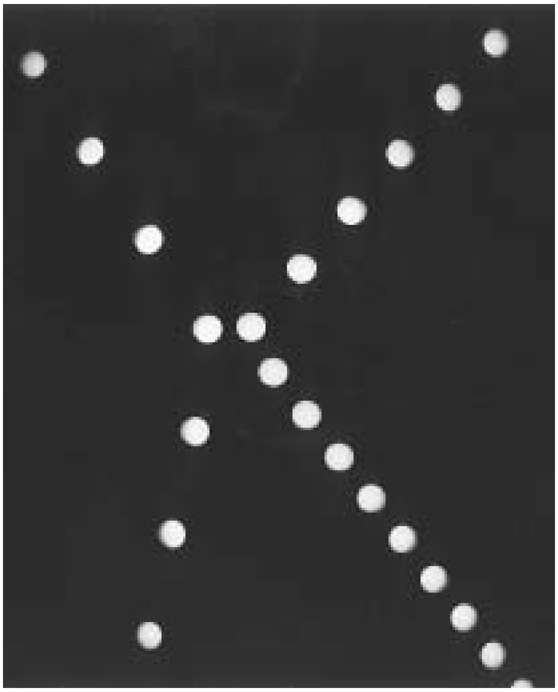


FIGURE 5.9 Stroboscopic photographs of two balls colliding. A ball enters from left top at a higher speed than the one from the right top. They collide near the center of the picture and then separate at different speeds.

These worked-out examples display a characteristic feature of physics: again and again, physics problems are solved by applying the expression of a *general* law to a specific situation. Both the beginning student and the veteran research physicist find it helpful, but also awesome, that one can *do* this. It seems strange that a few general laws enable one to solve an almost infinite number of specific individual problems. As Einstein expressed it in a letter to a friend:

> Even though the axioms of a theory are posed by human beings, the success of such an enterprise assumes a high degree of order in the objective world which one is not at all authorized to expect a priori. This is the wonder which is supported more and more with the development of our knowledge.*

Everyday life seems so very different. There you usually cannot calculate answers from general laws. Rather, you have to make quick decisions, some based on rational analysis, others based on "intuition." But the use of general laws to solve scientific problems becomes, with practice, quite natural also.

* A. Einstein to M. Solvine, letter of March 30, 1952.

5.4 MOMENTUM AND NEWTON'S LAWS OF MOTION

Earlier in this chapter we developed the concept of momentum and the law of conservation of momentum by considering experiments with colliding carts. The law was an "empirical" law; that is, it was discovered (perhaps "invented" or "induced" are better terms) as a generalization from experiment.

We can show, however, that the law of conservation of momentum also follows directly from Newton's laws of motion. It takes only a little algebra; that is, we can *deduce* the law from an established theory! Conversely, it is also possible to derive Newton's laws from the conservation law. Which of these is the fundamental law and which the conclusion drawn from it is therefore a bit arbitrary. Newton's laws used to be considered the fundamental ones, but since about 1900 the conservation law has been assumed to be the fundamental one.

Newton's second law expresses a relation between the net force $\mathbf{F}_{\text{net}}$ acting on a body, the mass m of the body, and its acceleration $\mathbf{a}$. We wrote this as $\mathbf{F}_{\text{net}} = m\mathbf{a}$. We can also write this law in terms of *change of momentum* of the body. Recalling that acceleration is the rate-of-change of velocity, $\mathbf{a} = \Delta\mathbf{v}/\Delta t$, we can write

$$\mathbf{F}_{\text{net}} = m\mathbf{a} = \frac{\Delta\mathbf{v}}{\Delta t},$$

or

$$\mathbf{F}_{\text{net}}\,\Delta t = m\,\Delta\mathbf{v}.$$

If the mass of the body is constant, the change in its momentum, $\Delta(m\mathbf{v})$, is the same as its mass times its change in velocity, $m(\Delta\mathbf{v})$, since only the velocity changes. Then we can write

$$\mathbf{F}_{\text{net}}\,\Delta t = \Delta(m\mathbf{v}).$$

That is, *the product of the net force on a body and the time interval during which this force acts equals the change in momentum of the body*. (The quantity $\mathbf{F}\,\Delta t$ is called the "impulse.")

This statement of Newton's second law is more nearly how Newton expressed it in his *Principia*. Together with Newton's third law, it enables us to derive the law of conservation of momentum for the cases we have studied. The details of the derivation are given in the *Student Guide*, "Deriving Conservation of Momentum from Newton's Laws." Thus, Newton's laws

A COLLISION IN TWO DIMENSIONS

The stroboscopic photograph shows a collision between two wooden disks on a frictionless horizontal table photographed from straight above the table. The disks are riding on tiny plastic spheres which make their motion nearly frictionless. Body B (marked ×) is at rest before the collision. After the collision it moves to the left, and Body A (marked −) moves to the right. The mass of Body B is known to be twice the mass of Body A: $m_B = 2m_A$. We will analyze the photograph to see whether momentum was conserved. (*Note:* The size reduction factor of the photograph and the [constant] stroboscopic flash rate are not given here. But as long as all velocities for this test are measured in the same units, it does not matter here what those units are.)

In this analysis, we will measure in centimeters the distance the disks moved on the photograph. We will use the time between flashes as the unit of time. Before the collision, Body A (coming from the lower part of the photograph) traveled 36.7 mm in the time between flashes: $\mathbf{v}_A$ = 36.7 speed-units. Similarly, we find that $\mathbf{v}'_A = 17.2$ speed-units, and $\mathbf{v}'_B = 11.0$ speed units.

The total momentum before the collision is just $m_A\mathbf{v}_A$. It is represented by an arrow 36.7 momentum-units long, drawn at right.

The vector diagram shows the momenta $m_A\mathbf{v}'_A$ and $m_B\mathbf{v}'_B$ after the collision; $m_A\mathbf{v}'_A$ is represented by an arrow 17.2 momentum-units long. Since $m_B = 2m_A$, the $m_B\mathbf{v}'_B$ arrow is 22.0 momentum-units long.

The dotted line represents the vector sum of $m_A\mathbf{v}'_A$ and $m_B\mathbf{v}'_B$, that is, the total momentum after the collision. Measure-

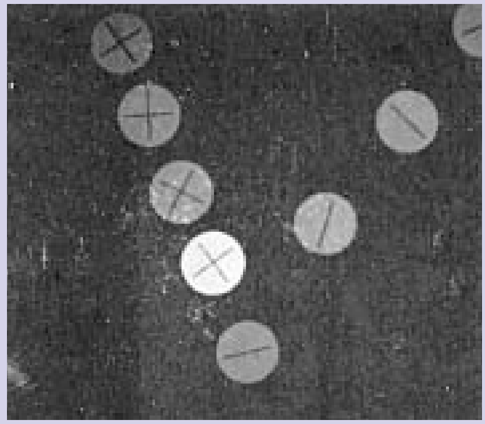


FIGURE 5.10

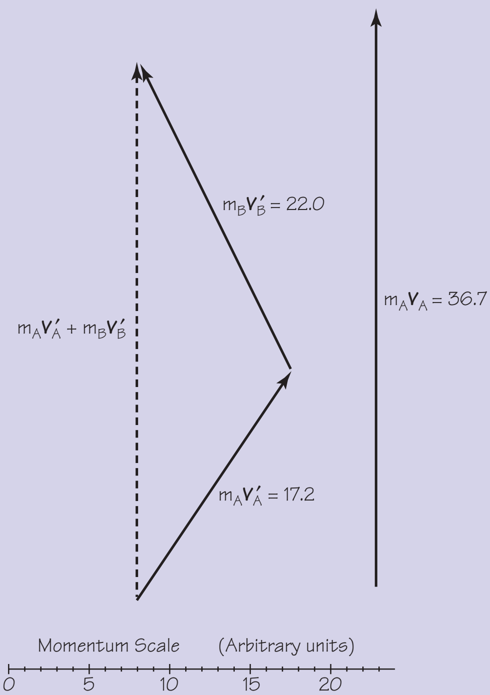


FIGURE 5.11 Momentum diagram of the two-dimensional collision pictured in Fig. 5.10.

ment shows it to be 34.0 momentum-units long. Thus, our measured values of the total momentum before and after the collision differ by 2.7 momentum-units. This is a difference of about 7%. We can also verify that the direction of the total is the same before and after the collision to within a small uncertainty.

Have we now demonstrated that momentum was conserved in the collision? Is the 7% difference likely to be due entirely to measurement inaccuracies? Or is there reason to expect that the total momentum of the two disks after the collision is really a bit less than before the collision?

and the law of conservation of momentum are not separate, independent laws of nature.

In all the examples considered so far and in the derivation above, we have considered each piece of the system to have a constant mass. But the definition of momentum permits a change of momentum to arise from a change of mass as well as from a change of velocity. In many cases, the mass of the object involved is in fact changing. For example, as a rocket spews out exhaust gases, its mass is decreasing; conversely, the mass of a train of coal cars increases as it rolls past a hopper that drops coal into the cars. The LCM remains valid for cases such as these, where the masses of the objects involved are not constant, as long as no net forces act on the system as a whole, and the momenta of all parts (including, say, that of the rocket's exhaust, are included).

One great advantage of being able to use the LCM is that it is a law of the kind that simply says "before = after." Thus, it applies in cases where you do not have enough information to use Newton's laws of motion during the whole interval between "before" and "after." For example, suppose a cannon that is free to move fires a shell horizontally. Although it was initially at rest, the cannon is forced to move while firing the shell; it *recoils*. The expanding gases in the cannon barrel push the cannon backward just as hard as they push the shell forward. You would need a continuous record of the magnitude of the force in order to apply Newton's second law separately to the cannon and to the shell to find their respective accelerations during their movement away from each other. A much simpler way is to use the LCM to calculate the recoil. The momentum of the system (cannon plus shell) is zero initially. Therefore, by the LCM, the momentum will also be zero after the shell is fired. If you know the masses of the shell and the cannon, and the speed of the emerging shell after firing, you can calculate the speed of the recoil (or the speed of the shell, if you measure the cannon's recoil speed). Moreover, if both speeds can be measured after the separation, then the ratio of the masses of the two objects involved can be calculated.

5.5 ISOLATED SYSTEMS

There are important similarities between the conservation law of mass and that of momentum. Both laws are tested by observing systems that may be considered to be isolated from the rest of the Universe. When testing or using the law of conservation of *mass*, an isolated system such as a sealed flask is used. Matter can neither enter nor leave this system. When testing or using the law of conservation of *momentum*, another kind of isolated system, one which experiences no net force from outside the system, is used.

Consider, for example, two frictionless carts colliding on a smooth horizontal table, or two hockey pucks colliding on smooth ice. The very low friction experienced by the pucks allows us to think away the ice on which they move, and to consider just the pucks to form a very nearly closed or isolated system. The table under the carts and the ice under the pucks do not have to be included since their individual effects on each of the objects cancel. That is, each puck experiences a downward gravitational force exerted by the Earth, while the ice on the Earth exerts an equally strong upward push.

Even in this artificial example, the system is not entirely isolated. There is always a little friction with the outside world. The layer of gas under the puck and air currents, for example, provide some friction. All outside forces are not *completely* balanced, and so the two carts or pucks do not form a truly isolated system. Whenever this is unacceptable, one can expand or extend the system so that it *includes* the bodies that are responsible for the external forces. The result is a new system on which the unbalanced forces are small enough to ignore.

For example, picture two automobiles skidding toward a collision on an icy road. The frictional forces exerted by the road on each car may be several hundred newtons. These forces are very small compared to the immense force (thousands of newtons) exerted by each car on the other when they collide. Thus, for many purposes, the action of the road can be ignored. For such purposes, the two skidding cars *before, during, and after the collision* are nearly enough an isolated system. However, if friction with the road (or the table on which the carts move) is too great to ignore, the law of conservation of momentum still holds, if we apply it to a larger system—one which includes the road or table. In the case of the skidding cars or the carts, the road or table is attached to the Earth. So the entire Earth would have to be included in a "closed system."

DERIVING CONSERVATION OF MOMENTUM FROM NEWTON'S LAWS

Suppose two bodies with masses m_A and m_B exert forces on each other (by gravitation or by magnetism, etc.). $\mathbf{F}_{AB}$ is the force exerted on body A by body B, and $\mathbf{F}_{BA}$ is the force exerted on body B by body A. No other unbalanced force acts on either body; they form an isolated system. By Newton's third law, the forces $\mathbf{F}_{AB}$ and $\mathbf{F}_{BA}$ are at every instant equal in magnitude and opposite in direction. Each body acts on the other for exactly the same time Δt. Newton's second law, applied to each of the bodies, says

$$\mathbf{F}_{AB}\,\Delta t = \Delta(m_A\mathbf{v}_A)$$

and

$$\mathbf{F}_{BA}\,\Delta t = \Delta(m_B\mathbf{v}_B).$$

By Newton's third law,

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA}$$

so that

$$\mathbf{F}_{AB}\,\Delta t = -\mathbf{F}_{BA}\,\Delta t.$$

Therefore,

$$\Delta(m_A\mathbf{v}_A) = -\Delta(m_B\mathbf{v}_B).$$

Suppose that the masses m_A and m_B are constant. Let $\mathbf{v}_A$ and $\mathbf{v}_B$ stand for the velocities of the two bodies at some instant,

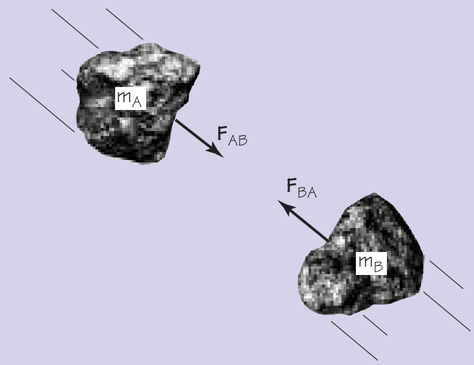


FIGURE 5.12 Collision between two rocks.

and let $\mathbf{v}'_A$ and $\mathbf{v}'_B$ stand for their velocities at some later instant. Then we can write the last equation as

$$m_A\mathbf{v}'_A - m_A\mathbf{v}_A = -(m_B\mathbf{v}'_B - m_B\mathbf{v}_B).$$

and

$$m_A\mathbf{v}'_A - m_A\mathbf{v}_A = -m_B\mathbf{v}'_B + m_B\mathbf{v}_B$$

A little rearrangement of terms leads to

$$m_A\mathbf{v}'_A + m_B\mathbf{v}'_B = m_A\mathbf{v}_A + m_B\mathbf{v}_B.$$

You will recognize this as our original expression of the law of conservation of momentum.

Here we are dealing with a system consisting of two bodies. This method works equally well for a system consisting of any number of bodies.

5.6 ELASTIC COLLISIONS

In 1666, members of the recently formed Royal Society of London witnessed a demonstration. Two hardwood balls of equal size were suspended at the ends of two strings, forming two pendula. One ball was released from rest at a certain height. It swung down and struck the other, which had been hanging at rest.

After impact, the first ball stopped at the point of impact while the second ball swung from this point, as far as one could easily observe, to the same height as that from which the first ball had been released. When the second ball returned and struck the first, it was now the second ball which stopped at the point of impact as the first swung up to almost the same height from which it had started. This motion repeated itself for several swings. (You can repeat it with a widely available desk toy.)

This demonstration aroused great interest among members of the Royal Society. In the next few years, it also caused heated and often confusing arguments. Why did the balls rise each time to nearly the same height after each collision? Why was the motion "transferred" from one ball to the

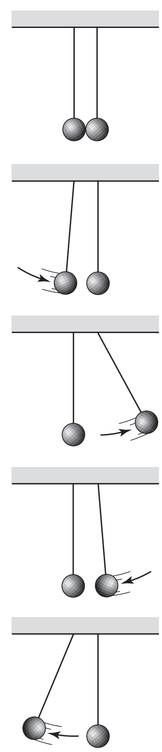


FIGURE 5.13 Demonstration with two pendula (similar to the demonstration witnessed by Royal Society members in 1666)

other when they collided? Why did the first ball not bounce back from the point of collision, or continue moving forward after the second ball moved away from the collision point?

The LCM explains what is observed, but it would also allow quite different results for different cases. The law says only that the momentum of ball A just before it strikes the resting ball B is equal to the total momentum of A and B just after collision. It does not say how A and B share the momentum. The actual result is just one of infinitely many different outcomes that would all agree with conservation of momentum. For example, suppose (though it has never been observed to happen) that ball A bounced back with ten times its initial speed. Momentum would still be conserved *if* ball B went on its way at 11 times A's initial speed.

In general symbols,

$\Delta\sum_i(\frac{1}{2}m_i\mathbf{v}_i^2) = 0.$

In 1668, three men reported to the Royal Society on the whole matter of impact. The three were the mathematician John Wallis, the architect and scientist Christopher Wren, and the physicist Christian Huygens. Wallis and Wren offered partial answers for some of the features of collisions; Huygens analyzed the problem in complete detail.

Huygens explained that in such collisions *another conservation law*, in addition to the law of conservation of momentum, also holds. Not only is the vector sum of the values of (mass × velocity) conserved, but so is the ordinary arithmetic sum—as we would now express it—of the values of $\frac{1}{2}mv^2$ for the colliding spheres! In modern algebraic form, the relationship he discovered can be expressed as

$$\tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = \tfrac{1}{2}m_A v_A'^2 + \tfrac{1}{2}m_B v_B'^2.$$

The quantity $\frac{1}{2}mv^2$—a scalar, not a vector—has come to be called *kinetic energy*, from the Greek word *kinetos*, meaning "moving." (The origin of the ½, which does not really affect the rule here, is shown in the *Student Guide* discussion for this chapter, "Doing Work on a Sled.") The equation stated above, then, is the mathematical expression of *the conservation of kinetic energy*. This relationship holds for the collision of two "perfectly hard" objects similar to those observed at the Royal Society meeting. There, ball A stopped and ball B went on at A's initial speed. This is the *only* result that agrees with *both* conservation of momentum and conservation of kinetic energy, as you can demonstrate yourself.

But is the conservation of kinetic energy as general as the law of conservation of momentum? Is the total kinetic energy present conserved in *any* interaction occurring in *any* isolated system?

It is easy to see that it is not, that it holds only in special cases such as that observed at the Royal Society test (or on making billiard ball colli-

FIGURE 5.14 Christian Huygens (1629–1695) was a Dutch physicist and inventor. He devised an improved telescope with which he discovered a satellite of Saturn and saw Saturn's rings clearly. Huygens was the first to obtain the expression for centripetal acceleration (v^2/R); he worked out a wave theory of light; and he invented a pendulum-controlled clock. Huygens' reputation would undoubtedly have been greater had he not been overshadowed by his contemporary, Newton.

sions). Consider the first example of Section 5.2. Two carts of equal mass (and with putty between the bumping surfaces) approach each other with equal speeds. They meet, stick together, and stop. The kinetic energy of the system after the collision is 0, since the speeds of both carts are zero. Before the collision the kinetic energy of the system was $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$. Both $\frac{1}{2}m_A v_A^2$ and $\frac{1}{2}m_B v_B^2$ are always positive numbers. Their sum cannot equal zero (unless both v_A and v_B are zero, in which case there would be no collision and not much of a problem). The kinetic energy of the system is *not* conserved in this collision in which the bodies stick together, while momentum *is* conserved. In fact, *no* collision in which the bodies stick together will show conservation of kinetic energy. It applies only to the collision of "perfectly hard" bodies that bounce back from each other.

The law of conservation of kinetic energy, then, is *not* as general as the law of conservation of momentum. If two bodies collide, the kinetic energy may or may not be conserved, depending on the type of collision. It *is* conserved if the colliding bodies do not crumple or smash or dent or stick together or heat up or change physically in some other way. Bodies that rebound without any such change are called *perfectly elastic*, whether they are billiard balls or subatomic particles. Collisions between them are called *perfectly elastic collisions*. In perfectly elastic collisions, *both* momentum and kinetic energy are conserved.

But most collisions are not perfectly elastic, and kinetic energy is not conserved. Thus, the sum of the $\frac{1}{2}m\mathbf{v}^2$ values after the collision is *less* than that before the collision. Depending on how much kinetic energy is "lost," such collisions might be called "partially elastic" or "perfectly inelastic." The loss of kinetic energy is greatest in perfectly inelastic collisions, when the colliding bodies remain together.

Collisions between steel ball bearings, glass marbles, hardwood balls, billiard balls, or some rubber balls (silicone rubber) are almost perfectly elastic, if the colliding bodies are not damaged in the collision. The total kinetic energy after the collision might be as much as, say, 96% of this value before the collision. Examples of perfectly elastic collisions are found only in collisions between atoms or subatomic particles. But all is not lost—we shall see how to deal with inelastic collisions also.

5.7 LEIBNIZ AND THE CONSERVATION LAW

Gottfried Wilhelm Leibniz (1646–1716) extended conservation ideas to phenomena other than collisions. For example, when a stone is thrown straight upward, its kinetic energy decreases as it rises, even without any

FIGURE 15.15 Gottfried Wilhelm Leibniz (1646–1716), a contemporary of Newton, was a German philosopher and diplomat and advisor to Louis XIV of France and Peter the Great of Russia. Independently of Newton, Leibniz invented the method of mathematical analysis called calculus. A long public dispute resulted between the two great men concerning the priority of ideas.

WHAT IS CONSERVED? THE DEBATE BETWEEN DESCARTES AND LEIBNIZ

René Descartes believed that the total quantity of motion in the Universe did not change. He wrote in his *Principles of Philosophy*:

> It is wholly rational to assume that God, since in the creation of matter He imparted different motions to its parts, and preserves all matter in the same way and conditions in which He created it, so He similarly preserves in it the same quantity of motion.

Descartes proposed to define the quantity of motion of an object as the product of its mass and its speed. As you saw in Section 5.3, this product is a conserved quantity only if there are no outside forces.

Gottfried Wilhelm Leibniz was aware of the error in Descartes' ideas on motion. In a letter in 1680 he wrote:

> M. Descartes' physics has a great defect; it is that his rules of motion or laws of nature, which are to serve as the basis, are for the most part false. This is demonstrated. And his great principle, that the quantity of motion is conserved in the world, is an error.

FIGURE 5.16 René Descartes (1596–1650) was the most important French scientist of the seventeenth century. In addition to his early contribution to the idea of momentum conservation, he is remembered by scientists as the inventor of coordinate systems and the graphical representation of algebraic equations. Descartes' system of philosophy, which used the deductive structure of geometry as its model, is still influential.

collision. At the top of the trajectory, kinetic energy is zero for an instant. Then it reappears and increases as the stone falls. Leibniz wondered whether something applied or given to a stone at the start is somehow *stored* as the stone rises, instead of being lost. His idea would mean that kinetic energy is just one part of a more general and really conserved quantity.

It was a hint that was soon followed up, with excellent results—once more an illustration of how science advances by successive innovators improving on partial truths.

5.8 WORK

In everyday language, pitching, catching, and running on the baseball field are "playing," while using a computer, harvesting in a field, or tending to an assembly line are "working." However, in the language of physics, "work" has been given a rather special definition, one that involves physical concepts of force and displacement instead of the subjective ones of reward or accomplishment. It is more closely related to the simple sense of effort or labor. The work done on an object is defined as the *product of the force exerted on the object times the displacement of the object along the direction of the force*. (You will see in Chapter 6 one origin of this definition in connection with the steam engine.)

When you move the hand and arm to throw a baseball, you exert a large force on it while it moves forward for about 1 m. In doing so, you (i.e., your muscles) do a large amount of work, according to the above definition. By contrast, in writing or in turning the pages of a book you exert only a small force over a short distance. This does not require much work, as the term "work" is understood in physics.

Suppose you have to lift boxes from the floor straight upward to a table

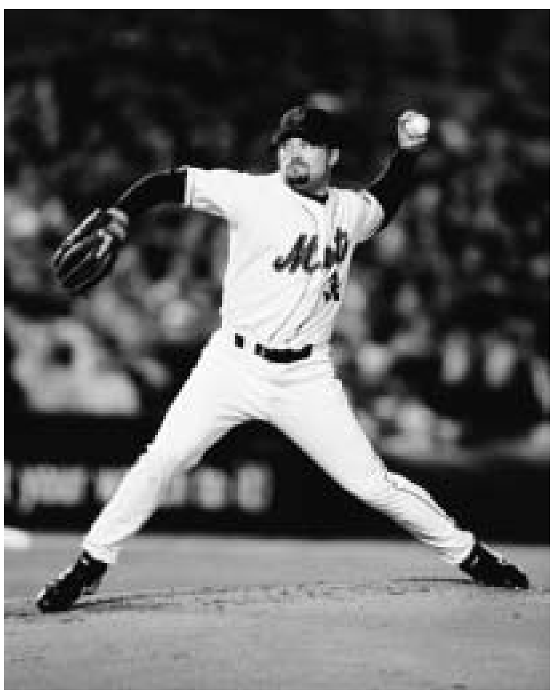


FIGURE 5.17 Major League baseball pitcher Mike Hampton.

at waist height. Here the language of common usage and that of physics both agree that you are doing work. If you lift two identical boxes at once, you do twice as much work as you do if you lift one box. If the table were twice as high above the floor, you would do twice as much work to lift a box to it. The work you do depends on both the *magnitude* of the force you must exert on the box and the *distance* through which the box moves in the direction of the force. Note that the work you do on a box does not depend on how long it takes to do your job.

We can now define the work W done on an object by a force $\mathbf{F}$ more precisely as the product of the magnitude F of the force and the distance d that the object moves *in the direction of* F while the force is being exerted; in symbols,

$$W = Fd$$

Note that work is a scalar quantity; it has only a magnitude but not a direction. As an example, while you are lifting a box weighing 100 N upward through 0.8 m you are applying a force of 100 N to the box. The work you

FIGURE 5.18

More generally, the definition of mechanical work is $W = Fd \cos \theta$, where θ is the angle between the vectors **F** and **d**. So, if $\theta = 90°$, $\cos \theta = 0$, and $W = 0$; if $\theta = 0°$, $\cos \theta = 1$, and $W = Fd$.

have done on the box to move it through the distance is $100 \text{ N} \times 0.8 \text{ m} = 80 \text{ N} \cdot \text{m}$.

From the definition of work, it follows that no work is done if there is no displacement. No matter how hard you push on a wall, no work is done if the wall does not move. Also, by our definition, no work is done if the only motion is perpendicular to the direction of the force. For example, suppose you are carrying a book bag. You must pull up against the downward pull of gravity to keep the bag at a constant height. But as long as you are standing still you do no work on the bag. Even if you walk along with it steadily in a horizontal line, the only work you do is in moving it forward against the small resisting force of the air.

5.9 WORK AND KINETIC ENERGY

Work is a useful concept in itself. The concept is most useful in understanding the concept of *energy*. There are a great many forms of energy, in addition to kinetic energy discussed in Section 5.6. A few of them will be discussed in this and succeeding chapters. We will define them, in the sense of describing how they can be measured and how they can be expressed algebraically. We will also discuss how energy changes from one form to another. The *general* concept of energy is difficult to define. But to define some *particular* forms of energy is easy enough. The concept of work helps greatly in making such definitions.

The chief importance of the concept of work is that work represents an amount of energy transformed from one form to another. For example, when you throw a ball you do work on it. While doing so, you transform chemical energy, which your body obtains from food and oxygen, into energy of motion of the ball. When you lift a stone (doing work on it), you transform chemical energy into what is called gravitational potential energy (discussed in the next section). If you release the stone, the Earth pulls it downward (does work on it); gravitational potential energy is transformed into kinetic energy. When the stone strikes the ground, it compresses the ground below it (does work on it), and its kinetic energy is transformed into heat and into work done to deform the ground on which it lands. In each case, the work is a measure of how much energy is transferred.

The form of energy called kinetic energy is the simplest to deal with. We can use the definition of work, $W = Fd$, together with Newton's laws of motion, to get an expression of this form of energy. Imagine that you exert a constant net force **F** on an object of mass m. This force accelerates the object over a distance **d** in the same direction as **F** from rest to a speed v. Using Newton's second law of motion, we can show that

$$Fd = \tfrac{1}{2}mv^2.$$

(The details of this derivation are given in the *Student Guide*, "Doing Work on a Sled.")

Fd is the expression for the work done on the object by whatever agency exerted the force **F**. The work done on the object equals the amount of energy transformed from some form into the energy of motion, the kinetic energy, of the object. The symbol KE is often used to represent kinetic energy. By definition, then

$$KE = \tfrac{1}{2}mv^2.$$

The expression $\tfrac{1}{2}mv^2$ relates directly to the concept of work and so provides a useful expression for the energy of motion.

The equation $Fd = \tfrac{1}{2}mv^2$ was obtained by considering the case of an object initially at rest. In other words, the object had an initial kinetic energy of zero. The relation can be extended to hold also for an object already in motion when the net force is applied (e.g., a bat hitting a moving ball). In that case, the work done on the object still equals the change in its kinetic energy from its initial to its final value

$$Fd = \Delta(KE).$$

The quantity $\Delta(KE)$ is, by definition, equal to $(\tfrac{1}{2}mv^2)_{\text{final}} - (\tfrac{1}{2}mv^2)_{\text{initial}}$.

Work is defined as the product of a force and a distance. Therefore, its units in the mks system are *newtons* × *meters* or newton-meters: A newton-meter is given a special name. It is also called a *joule* (symbol J) in honor of James Prescott Joule, the nineteenth-century physicist famous for his experiments showing that heat is a form of energy (Chapter 6). The joule is the unit of work or of energy, when force is measured in newtons and distance in meters. When force is measured in dynes and distance in centimeters, the unit of work or energy is *dynes* × *centimers*. A dyne-centimeter is also given a special name: *erg*.

5.10 POTENTIAL ENERGY

As you saw in the previous section, doing work on an object can increase its kinetic energy. Work also can be done on an object *without* increasing its kinetic energy. For example, while you lifted that box up to the table in

Section 5.8 at a small, constant speed, kinetic energy remains constant. But you were doing work on the box. By doing work you are using your body's store of chemical energy. Into what form of energy is it being transformed?

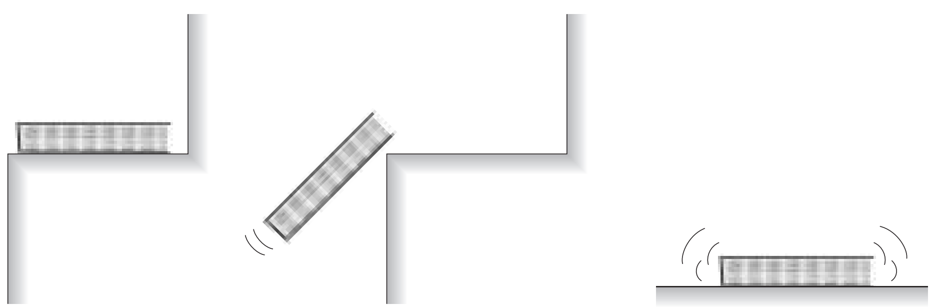
The answer, as Leibniz suggested, is that there is "energy" somehow associated with height above the Earth. This energy is now called *gravitational potential energy*. Lifting a box or a book higher and higher increases the gravitational potential energy associated with the lifted object. You can see clear evidence of this effect when, say, you pick up a book from the floor, lift it to a certain height, and then let it drop. The gravitational potential energy is transformed rapidly into the kinetic energy of a fall. In general terms, suppose a force F is used to displace an object upward a distance d, without changing its KE. Then, the increase in gravitational potential energy, symbolized by $\Delta(PE)_{\text{grav}}$, is

$$\Delta(PE)_{\text{grav}} = F_{\text{applied}} \cdot d$$

$$= -F_{\text{grav}} \cdot d.$$

Potential energy can be thought of as *stored* energy. As the book falls, its gravitational potential energy decreases while its kinetic energy increases correspondingly. When the book reaches its original level, all of the gravitational potential energy stored during the lift will have been transformed into kinetic energy.

Many useful applications follow from this idea of potential or stored energy. For example, a steam hammer used by construction crews is driven upward by high-pressure steam (thus gaining potential energy). When the steam supply stops, the hammer drops, the gravitational potential energy is converted into kinetic energy. Another example is the use of energy from electric power plants during low-demand periods to pump water into a high reservoir. When there is a large demand for electricity later, the water is



(a) (b) (c)

FIGURE 5.19 Object (a book) at rest, falling, and coming to rest again.

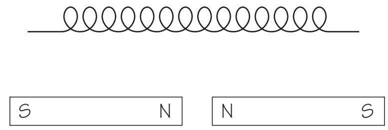


FIGURE 5.20 Two forms of potential energy (in an extended spring and in a pair of magnets).

allowed to run down and drive the electric generators, recouping the stored energy for use.

There are forms of potential energy other than gravitational. For example, if you stretch a rubber band or a spring, you increase its *elastic potential energy*. When you release the rubber band, it can deliver the stored energy to a projectile in the form of kinetic energy. Nearly all of the work done in blowing up an elastic balloon is also stored as potential energy.

Other forms of potential energy are associated with other kinds of forces. In an atom, the negatively charged electrons are attracted by the positively charged nucleus. If an externally applied force pulls an electron *away* from the nucleus, the *electric potential energy* increases. If, then, the electron is freed of the applied forces and moves back *toward* the nucleus, the potential energy decreases as the electron's kinetic energy increases. Or, if two magnets are pushed together with north poles facing, the *magnetic potential energy* increases. When released, the magnets will move apart, gaining kinetic energy as they lose potential energy.

Where is the potential energy located in all these cases? It is easy to think at first that it "belongs" to the body that has been moved. But without the presence of the other object (the Earth, the nucleus, the other magnet) neither would work be needed for steady motion, nor would there be

FIGURE 5.21 Chemical energy (ultimately from the Sun) is stored in rice and converted into work by the farmer who cultivates the crop.

any increase in potential energy. Rather, action on the object would increase only the kinetic energy of the object on which work was done. We must conclude that the potential energy belongs not to *one* body, but to the whole system of interacting bodies involved! This is evident in the fact that the potential energy gained is available to any one or to all of these interacting bodies. For example, either of the two magnets would gain all the kinetic energy just by releasing it and holding the other in place. Or suppose you could fix the book somehow to a hook in space that would hold it fixed there. The Earth would then "fall" up toward the book, being attracted to it. Eventually the Earth would gain just as much kinetic energy at the expense of stored potential energy as the book would if it were free to fall to the Earth.

The increase in gravitational potential energy "belongs" to the Earth–book *system*, not the book alone. The work is done by an "outside" agent (you), increasing the total energy of the Earth–book system. When the book falls, it is responding to forces exerted by one part of the system on another. The *total energy* of the system does not change; it is converted from *PE* to *KE*. This is discussed in more detail in the next section.

In fact, during the fall of the book, as initially postulated, the Earth would be moving a little toward the falling book. Why don't we observe this?

5.11 CONSERVATION OF MECHANICAL ENERGY

In Section 5.9, you learned that the amount of work done on an object is *equal* to the amount of energy transformed from one form to another. For example, the chemical energy of muscles is transformed into the kinetic energy of a thrown ball. The work you have done while throwing the ball is equal to the energy you have given up from your store of chemical energy. This statement in the first sentence implies that the *amount* of energy involved during an interaction does not change; only its *form* changes. This is particularly obvious in motions where no "outside" force is applied to a mechanical system.

While a stone falls freely, for example, the gravitational potential energy of the stone–Earth system is continually transformed into kinetic energy. Neglecting air friction, the *decrease* in gravitational potential energy is, for any portion of the path, equal to the *increase* in kinetic energy. Consider a stone thrown upward, which we will call the positive direction. Between any two points in its path, the *increase* in gravitational potential energy equals the *decrease* in kinetic energy. Consider a stone rising upward after being thrown in the upward direction. After it leaves your hand, the only force applied is F_{grav} (neglect external forces such as friction). The gravitational force is pointing downward in the negative direction. The work done by this force (with d positive for upward displacements) is

$$-F_{grav}\,d = \Delta(PE)_{grav}$$

$$= -\Delta KE.$$

In words, these equations state that the work done by gravity as the stone rises against it results in an increase in potential energy but a decrease in kinetic energy—the stone slows down. This relationship can be rewritten as

$$\Delta(KE) + \Delta(PE)_{grav} = 0,$$

or, still more concisely, as

$$\Delta(KE + PE_{grav}) = 0.$$

If $(KE + PE_{grav})$ represents the *total mechanical energy* of the system (consisting here of the stone and the Earth), then the *change* in the system's total mechanical energy is *zero*, *provided there is no outside work added to the system (e.g., a strong wind acting on the stone)*. In other words, when outside work is zero, the total mechanical energy of a system, $(KE + PE_{grav})$, remains constant; it is *conserved*.

A similar statement can be made for a vibrating guitar string. While the string is being pulled away from its unstretched position, the string–guitar system gains elastic potential energy. When the string is released, the elastic potential energy decreases while the kinetic energy of the string increases. The string coasts through its unstretched position and becomes stretched in the other direction. Its kinetic energy then decreases as the elastic potential energy increases. As it vibrates, there is a repeated transformation of elastic potential energy into kinetic energy and back again. The string loses some mechanical energy; for example, sound waves radiate away. Otherwise, the decrease in elastic potential energy over any part of the string's motion would be accompanied by an equal increase in kinetic energy, and vice versa:

> The equations in this section are valid only if friction is negligible. We shall extend the range later to include friction, which can cause the conversion of mechanical energy into heat energy.

$$\Delta(PE)_{elastic} = -\Delta(KE),$$

FIGURE 5.22

or

$$-\Delta(PE)_{\text{elastic}} = \Delta(KE).$$

In such an ideal case, the total mechanical energy ($KE + PE_{\text{elastic}}$) remains constant; it is conserved.

Galileo's experiment with the pendulum (Section 3.1) can also be described in these terms. The gravitational potential energy is determined by the height to which the pendulum was originally pulled. That potential energy is converted to kinetic energy at the bottom of the swing and back to potential energy at the other side. Since the pendulum retains its initial energy, it will stop temporarily ($KE = 0$, $PE =$ max) only when it returns to its initial height.

Adding Work

You have seen that the potential energy of a system can be transformed into the kinetic energy of some part of the system, and vice versa. Suppose that an amount of work W is done on part of the system by some external force. Then the energy of the system is increased by an amount equal to W. Consider, for example, a suitcase–Earth system. You must do work on the suitcase to pull it away from the Earth up to the second floor. This work increases the total mechanical energy of the Earth–suitcase system. If

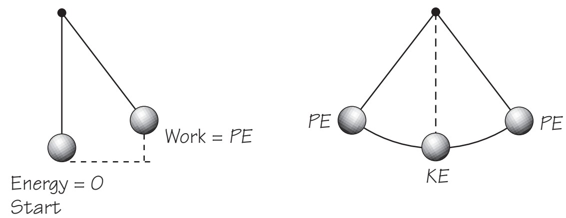


FIGURE 5.23 Work, kinetic, and potential energy in a swinging pendulum.

you yourself are included in the system, then your internal chemical energy decreases in proportion to the work you do. Therefore, the *total* energy of the lifter + suitcase + Earth system does not change, though of course the energy is redistributed among the parts of the system.

Relationship to Newton's Laws of Motion

The law of conservation of energy can be derived from Newton's laws of motion (although Newton himself did not do this). Therefore, it tells nothing that could not, in principle, be computed directly from Newton's laws of motion. However, there are situations where there is simply not enough information about the forces involved to apply Newton's laws, or where it would be inconvenient to try to measure them. It is in these cases that the law of conservation of mechanical energy demonstrates its maximum usefulness.

A perfectly elastic collision is a good example of a situation where we often cannot simply apply Newton's laws of motion. For such collisions, we cannot easily measure the force that one object exerts on the other. We do know that during the actual collision among perfectly elastic bodies, the objects distort one another. The distortions are produced against elastic forces. Thus, some of the combined kinetic energy of the objects is transformed into elastic potential energy as they distort one another. Then elastic potential energy is transformed back into kinetic energy as the objects separate. In an ideal case, both the objects and their surroundings are exactly the same after colliding as they were before.

However, the law of conservation of mechanical energy involves only the *total* energy of the objects before and after the collision. It does not try to give the kinetic energy of each object separately. So it is incomplete knowledge, but useful nevertheless. You may recall that the law of conservation of momentum also supplies incomplete but useful knowledge. It can be used to find the *total* momentum of elastic objects in collision, but not the *individual* momentum vectors. In Section 5.6 you saw how conservation of momentum and conservation of mechanical energy *together* limit the possible outcomes of perfectly elastic collisions. (As often in physics, two or

FIGURE 5.24 During its contact with a golf club, a golf ball is distorted. As the ball moves away from the club, and while the ball recovers its normal spherical shape, its elastic potential energy is transformed into (additional) kinetic energy.

more laws together show the limits of what science can say about phenomena.) For two colliding objects, these two restrictions are enough to give an exact solution for the two velocities after collision. For more complicated systems, conservation of energy remains important. Scientists usually are not interested in the detailed motion of every part of a complex system. They are not likely to care, for example, about the motion of every molecule in a rocket exhaust. Rather, they may want to know only about the exhaust's overall thrust and temperature. These can be found from the overall conservation laws.

5.12 FORCES THAT DO NO WORK

In Section 5.8, the *work* done on an object was defined as the product of the magnitude of the force **F** applied to the object and the magnitude of the distance **d** in the direction of **F** through which the object moves while the force is being applied. In all the examples so far, the object moved in the same direction as that of the force vector.

Usually, the direction of motion and the direction of the force are *not* the same. Let us revisit the example we briefly mentioned before (end of Section 5.8). Suppose you carry a book bag at constant speed horizontally, so that its kinetic energy does not change. Since there is no change in the book bag's energy, you are doing no work on it (by the definition of work). You do apply a force on the book bag, and the bag does move through a distance. But here the applied force and the distance the bag moves are at right angles. You exert a vertical force on the bag upward to balance its weight. But it moves horizontally, with you. Here, an applied force **F** is ex-

FIGURE 5.25 A balanced rock (Mojave Desert): potential energy waiting to be converted.

erted on an object while the object moves at right angles to the direction of the force. Therefore, **F** has no component in the direction of **d** and so the force *does no work*. This statement agrees entirely with the idea of work in physics as *energy being transformed from one form to another*. Since the book bag's speed is constant, its kinetic energy is constant. Since its distance from the Earth is constant, its gravitational potential energy is constant. Therefore, there is no transfer of mechanical energy. (Nevertheless, your arm does tire as you carry the book bag horizontally. The reason for this is that muscles are not rigid. They are constantly relaxing and tightening up again. This requires chemical energy, even though no work is done on the bag.)

A similar reasoning, but not so obvious, applies to a satellite in a circular orbit. The speed and the distance from the Earth are both constant. Therefore, the kinetic energy and the gravitational potential energy are both constant, and there is no energy transformation. For a circular orbit, the centripetal force vector is perpendicular to the tangential direction of motion at any instant. No work is being done on the satellite. But to put an artificial satellite into a circular orbit to start with requires work. Once it is in orbit, however, the *KE* and *PE* stay constant, and no further work is done on the satellite.

However, if the satellite's orbit is eccentric, the force vector is generally not perpendicular to the direction of motion. In such cases, energy is continually transformed between kinetic and gravitational potential forms. The total energy of the system (satellite and Earth) remains of course constant.

Situations where the net force is exactly perpendicular to the motion are as rare as situations where the force and motion are in exactly the same direction. What about the more usual case, involving some angle between the force and the motion?

In general, the work done on an object depends on how far the body moves *in the direction of the force*. As stated before, the equation $W = Fd$ properly defines work only if d is the distance the body moves in the direction of the force. The gravitational force is directed *down*. So only the distance *down* determines the amount of work done by $\mathbf{F}_{grav}$. Change in gravitational potential energy depends *only* on change in height, at least near the Earth's surface. For example, consider raising a suitcase from the first floor to the second floor of a building. The same increase in PE_{grav} of the suitcase–Earth system occurs regardless of the path by which the suitcase is raised. Also, each path requires the same amount of work.

More generally, change in PE_{grav} depends only on change of position. The details of the path followed in making the change make no difference at all. The same is true for changes in elastic potential energy, electric potential energy, etc. The changes depend only on the initial and final positions, and not on the path taken between these positions.

An interesting conclusion follows from the fact that change in PE_{grav} depends only on change in height. For example, consider a child on a slide. Starting from the top position, the gravitational potential energy decreases as his/her altitude decreases. If frictional forces are vanishingly small, all the work the Earth's pull does on him/her goes into transforming PE_{grav} into KE. Therefore, the increases in KE depend only on the decreases in altitude. In other words, the child's speed when he/she reaches the ground will be the same (absent friction) whether he/she slides down or jumps off the top.

A similar principle holds for satellites in orbit and for electrons in TV tubes. In the absence of losses to parts outside the system, the change in kinetic energy depends only on the initial and final positions, and not on

FIGURE 5.26 The change in kinetic energy depends only on the initial and final positions, and not on the path taken between them.

the path taken between them. This principle gives great simplicity to the use of physical laws, as you will see when you study gravitational and electric fields in Chapter 10.

SOME NEW IDEAS AND CONCEPTS

conservation law
elastic collision
isolated system
joule
kinetic energy
law of conservation of kinetic energy
law of conservation of mass
law of conservation of mechanical energy
law of conservation of momentum (LCM)
mechanical energy
momentum
Newtonian world machine
potential energy
system
work

FURTHER READING

F.L. Holmes, *Lavoisier and the Chemistry of Life* (Madison: University of Wisconsin Press, 1989).
G. Holton and S.G. Brush, *Physics, The Human Adventure* (Piscataway, NJ: Rutgers University Press, 2001), Chapters 15–18.
R.S. Westfall, *The Construction of Modern Science: Mechanisms and Mechanics* (New York: Cambridge University Press, 1978).

STUDY GUIDE QUESTIONS

5.1 Conservation of Mass

1. Why did scientists look for conservation laws?
2. True or false: Mass is conserved in a closed system only if there is no chemical reaction in the system.
3. If 50 cm^3 of alcohol is mixed with 50 cm^3 of water, the mixture amounts to only 98 cm^3. An instrument pack weighs much less on the Moon than on Earth. Are these examples of contradictions to the law of conservation of mass?
4. Which one of the following statements is true?
   (a) Lavoisier was the first person to believe that the amount of material in the Universe does not change.
   (b) Mass is measurably increased when heat enters a system.
   (c) A closed system was used to establish the law of conservation of mass experimentally.

5. Five grams (5 g) of a red fluid at 12°C having a volume of 4 ml are mixed in a closed bottle with 10 g of a blue fluid at 5°C having a volume of 8 ml. On the basis of this information only, what can you be sure of about the resulting mixture?

5.2 Collisions

1. Descartes defined the quantity of motion of an object as the product of its mass and its speed. Is his quantity of motion conserved as he believed it was? If not, how would you modify his definition so the quantity of motion would be conserved?
2. Two carts collide head-on and stick together. In which of the following cases will the carts be at rest immediately after the collision?

	Cart A		*Cart B*	
	Mass (kg)	*Speed before (m/s)*	*Mass (kg)*	*Speed before (m/s)*
(a)	2	3	2	3
(b)	2	2	3	3
(c)	2	3	3	2
(d)	2	3	1	6

5.3 Conservation of Momentum

1. State the law of conservation of momentum in terms of
   (a) a change in the total momentum of a system;
   (b) the total initial momentum and final momentum;
   (c) the individual parts of a system.
2. Under what condition is the law of conservation of momentum valid?
3. Which of the following has the least momentum? Which has the greatest momentum?
   (a) a pitched baseball;
   (b) a jet plane in flight;
   (c) a jet plane taxiing toward the terminal.
4. A girl on ice skates is at rest on a horizontal sheet of smooth ice. As a result of catching a rubber ball moving horizontally toward her, she moves at 2 cm/s. Give a rough estimate of what her speed would have been:
   (a) if the rubber ball were thrown twice as fast;
   (b) if the rubber ball had twice the mass;
   (c) if the girl had twice the mass;
   (d) if the rubber ball were not caught by the girl, but bounced off and went straight back with no change of speed.
5. A boy and a girl are on ice skates at rest near each other. The boy throws a ball to the girl in a straight line. Does he move? If so, in what way and why?

After she catches the ball, she throws it back to him in a high arc. Does she move? If so, in what way and why?

5.4 Momentum and Newton's Laws of Motion

1. Since the law of conservation of momentum can be derived from Newton's laws, what good is it?
2. Explain why a cannon shooting a cannon ball must experience a recoil.
3. What force is required to change the momentum of an object by 50 kg m/s in 15 s?

5.5 Isolated Systems

1. Define what is meant by a "closed" or "isolated" system for the purpose of the law of conservation of mass; for the purpose of the law of conservation of momentum.
2. Explain whether or not each of the following can be considered an isolated system:
   (a) a baseball thrown horizontally, after it leaves the thrower's hand;
   (b) a space shuttle orbiting the Earth;
   (c) the Earth and the Moon.
3. Three balls in a closed system have the following masses and velocities:
   ball A: 4 kg, 8 m/s left;
   ball B: 10 kg, 3 m/s up;
   ball C: 8 kg, 4 m/s right.
   Using the principles of mass and momentum conservation, what can you discover about the final condition of the system after the balls have come to rest in the system? What cannot be discovered?

5.6 Elastic Collisions

1. Which phrases correctly complete the statement? Kinetic energy is conserved:
   (a) in all collisions;
   (b) whenever momentum is conserved;
   (c) in some collisions;
   (d) when the colliding objects are not too hard.
2. Under what condition does the law of conservation of kinetic energy hold?
3. Explain why the conservation laws of kinetic energy and momentum are both sometimes needed to describe the outcome of a collision of two bodies.
4. Is the law of conservation of kinetic energy as general as the law of conservation of momentum? Explain.
5. Kinetic energy is never negative because:
   (a) scalar quantities are always positive;
   (b) it is impossible to draw vectors with negative length;
   (c) speed is always greater than zero;
   (d) kinetic energy is proportional to the square of the speed.

5.7 Leibniz and the Conservation Law

1. How would Leibniz have explained the apparent disappearance of the quantity $\frac{1}{2}m\mathbf{v}^2$ in the following situations?
   (a) during the upward motion of a thrown object;
   (b) when the object strikes the ground.
2. Give an example of a situation in which momentum is conserved but kinetic energy is not conserved.

5.8 Work

1. If a force of magnitude F is exerted on an object while the object moves a distance d in the direction of the force, the work done on the object is:
   (a) F; (b) Fd; (c) F/d; (d) $\frac{1}{2}Fd^2$.
2. Give two examples of situations in which a force is exerted on an object but no work is done.

5.9 Work and Kinetic Energy

1. The kinetic energy of a body of mass m moving at a speed v is given by the expression:
   (a) $\frac{1}{2}mv$; (b) $\frac{1}{2}mv^2$; (c) mv^2; (d) $2mv^2$; (e) m^2v^2.
2. What is the general relationship between work and energy?
3. You lift a book from the floor and put it on a shelf.
   (a) What happens to the work that you put into lifting the book?
   (b) Can the work ever be turned into kinetic energy? Explain how.

5.10 Potential Energy

1. Name some forms of potential energy. Describe a situation for each of these in which the potential energy would be greater than zero.
2. A stone of mass m falls a vertical distance d, pulled by its weight $F_{grav} = mg$, where g is the acceleration of gravity. The decrease in gravitational potential energy during the fall is:
   (a) md; (b) mg; (c) mgd; (d) $\frac{1}{2}md^2$; (e) d.
3. When you compress a coil spring, you do work on it. The elastic potential energy:
   (a) disappears; (b) breaks the spring; (c) increases; (d) decreases.
4. Two electrically charged objects repel one another. To increase the electric potential energy, you must:
   (a) make the objects move faster;
   (b) move one object in a circle around the other object;
   (c) attach a rubber band to the objects;
   (d) pull the objects farther apart;
   (e) push the objects closer together.
5. A pendulum bob is swinging back and forth. Where is the kinetic energy of the bob the greatest? Where is it the least?

5.11 Conservation of Mechanical Energy

1. As a stone falls frictionlessly:
   (a) its kinetic energy is conserved;
   (b) its gravitational potential energy is conserved;
   (c) its kinetic energy changes into gravitational potential energy;
   (d) no work is done on the stone;
   (e) there is no change in the total energy.
2. In which position is the elastic potential energy of a vibrating guitar string greatest? In which position is its kinetic energy greatest?
3. If a guitarist gives the same amount of elastic potential energy to a bass string and to a treble string, which one will gain more speed when released? (The mass of 1 m of bass string is greater than that of 1 m of treble string.)
4. Describe the changes in kinetic energy and potential for the system of the two colliding pendula observed at the Royal Society, as described in Section 5.6.

5.12 Forces That Do No Work

1. How much work is done on a satellite during each revolution if its mass is m, its period is T, its speed is v, and its orbit is a circle of radius R?
2. Two skiers were together at the top of a hill above a ski jump. While one skier skied down the slope and went off the jump, the other had a change of mind and rode the ski lift back down. Compare their changes in gravitational potential energy.
3. A third skier went directly down a straight slope next to the ski jump. How would this skier's speed at the bottom compare with that of the skier who went off the jump?
4. No work is done (select one):
   (a) on a heavy box when it is pushed at constant speed along a rough horizontal floor;
   (b) on a nail when it is hammered into a board;
   (c) when there is no component of force parallel to the direction of motion;
   (d) when there is no component of force perpendicular to the direction of motion.

DISCOVERY QUESTIONS

1. In the examples in this chapter we carefully neglected the effects of friction and air resistance. How would friction and air resistance affect the mechanical energy of a pendulum or a flying baseball? How would they affect the conservation of momentum?
2. A child is swinging on a swing. She asks you to push her higher.
   (a) Using the concepts in this chapter, explain why pushing her makes her go higher.
   (b) What is the best part of the swing to exert a push? Why?

3. Using the laws of conservation of momentum and energy, explain why cyclists should wear helmets and football players should wear padding.
4. Discuss the conversion between kinetic and potential forms of energy in the system of a planet orbiting the Sun.
5. Furniture movers are paid for their work in moving furniture. In which of the following cases is work, as defined in physics, *not* done by them or the truck on the furniture?
   (a) A piano is carried downstairs.
   (b) The piano is lifted onto a truck.
   (c) The truck accelerates to 50 mi/hr in 30 s.
   (d) The truck hits an ice patch and skids at constant speed for 2 s.
   (e) The truck hits an obstacle and comes to rest.
6. (a) Why can ocean liners or airplanes not turn corners sharply?
   (b) In the light of your knowledge of the relationship between momentum and force, comment on reports about so-called unidentified flying objects (UFOs) turning sharp corners in full flight.
7. The philosopher John Locke (1632–1704) proposed a science of human nature that was strongly influenced by Newton's physics. In Locke's atomistic view, elementary ideas ("atoms") are produced by elementary sensory experiences and then drift, collide, and interact in the mind. Thus, the formation of ideas was only a special case of the universal interactions of particles.

   Does such an approach to the subject of human nature seem reasonable to you? What argument for and against this sort of theory can you think of?

Quantitative

1. A person who lifts a 10-N book a distance d of 0.8 m straight up does 8 J of work. How much work would the person do if the book is lifted up 1.6 m?
2. The kinetic energy of a ball on a tabletop increases from 10 J to 20 J. How much work is done on it?
3. One joule (1 J) of work is put into lifting a pendulum bob from the zero-energy position. After it is let go, the kinetic energy at one point was found to be 0.25 J. What is the potential energy at that point?
4. A freight car of mass 10^5 kg travels at 2.0 m/s and collides with a motionless freight car of mass 1.5×10^5 kg on a horizontal track. The two cars lock and roll together after impact.
   (a) Using the law of conservation of momentum, find the velocity of the two cars after collision.
   (b) Using the result from (a), find the total kinetic energy of the two cars after the collision, and compare it with the total kinetic energy before the collision. Is this an example of an elastic collision?
   (c) Which quantities are conserved in this collision, and which are not?
5. A 1-kg billiard ball moving on a pool table at 0.8 m/s collides head-on with the cushion along the side of the table. The collision can here be regarded as perfectly elastic. What is the momentum of the ball:
   (a) before impact?

(b) after impact?
(c) What is the change in momentum of the ball?
(d) Is the momentum of the ball conserved?
(e) Is the kinetic energy of the ball conserved?
(f) If the duration of the collision is 0.01 s, what average force does the cushion experience?

6. A system consists of three elastic bodies with masses of 4 g, 6 g, and 8 g. They are squeezed together at rest at a single point and then released. They fly away from each other under the influence of the elastic forces, which are assumed equal for each. The 4-g body is moving with a velocity of 20 cm/s north, and the 6-g body is moving at 3 cm/s east. What is the velocity of the 8-g body?
7. Calculate the kinetic energy of a car and driver traveling at 100 km/hr (about 60 mi/hr). The mass of the car and driver is about 1000 kg. How did they obtain this kinetic energy? Where does it go when the driver puts on the brakes and comes to a stop? Where does it go if the car collides with another?