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CHAPTER 10

Electricity and Magnetism

10.1 GILBERT'S *MAGNETS*

Two natural substances, amber and lodestone, have awakened curiosity since ancient times. Amber is sap that oozed long ago from certain softwood trees, such as pine. Over many centuries, it hardened into a semitransparent solid akin to model plastics and ranging in color from yellow to brown. It is a handsome ornamental stone when polished, and sometimes contains the remains of insects that were caught in the sticky sap. Ancient Greeks recognized a strange property of amber. If rubbed vigorously against cloth, it can attract nearby objects, such as bits of straw or grain seeds.

Lodestone is a metallic mineral that also has unusual properties. It attracts iron. Also, when suspended or floated, a piece of lodestone always turns to one particular position, a north–south direction. The first known written description of the navigational use of lodestone as a compass in Western countries dates from the late twelfth century, but its properties were known even earlier in China. Today, lodestone would be called magnetized iron ore.

The histories of lodestone and amber are the early histories of magnetism and electricity. The modern developments in these subject areas began in 1600 with the publication in London of William Gilbert's book *De Magnete (On Magnets)*. Gilbert (1544–1603) was an influential physician, who served as Queen Elizabeth's chief physician. During the last 20 years of his life, he studied what was already known of lodestone and amber. Gilbert made his own experiments to check the reports of other writers and summarized his conclusions in *De Magnete*. The book is a classic in scientific literature, primarily because it was a thorough and largely successful attempt to test complex speculation by detailed experiment.

Gilbert's first task in his book was to review and criticize what had previously been written about lodestone. He discussed various theories about the cause of magnetic attraction. When it was discovered that lodestone and magnetized needles or iron bars tend to turn in a north–south direction, many authors offered explanations. But, says Gilbert:

> they wasted oil and labor, because, not being practical in the research of objects of nature, being acquainted only with books . . . they constructed certain explanations on the basis of mere opinions.

FIGURE 10.1 William Gilbert (1544–1603).

As a result of his own researchers, Gilbert himself proposed the real cause of the lining-up of a suspended magnetic needle or lodestone: The Earth itself is a lodestone and thus can act on other magnetic materials. Gilbert performed a clever experiment to show that his hypothesis was a likely one. Using a large piece of natural lodestone in the shape of a sphere, he showed that a small magnetized needle placed on the surface of the lodestone acts just as a compass needle does at different places on the Earth's surface. (In fact, Gilbert called his lodestone the *terrella*, or "little Earth.") If the directions along which the needle lines up are marked with chalk on the lodestone, they form meridian circles. Like the lines of equal longitude on a globe of the Earth, these circles converge at two opposite ends that may be called "poles." At the poles, the needle points perpendicular to the surface of the lodestone. Halfway between, along the "equator," the needle lies along the surface. Small bits of iron wire, when placed on the spherical lodestone, also line up in these same directions.

Nowadays, discussion of the actions of magnets generally involves the idea that magnets set up "fields" all around themselves, as further discussed in Section 10.3. The field can act on other objects, near or distant. Gilbert's description of the force exerted on the needle by his spherical lodestone was a step toward the modern field concept:

> The terrella's force extends in all directions. . . . But whenever iron or other magnetic body of suitable size happens within its sphere of influence it is attracted; yet the nearer it is to the lodestone the greater the force with which it is borne toward it.

Gilbert also included a discussion of electricity in his book. He introduced the word *electric* as the general term for "bodies that attract in the same way as amber." (The word *electric* comes from the Greek word *electron*, which means "amber." Today the word *electron* refers to the smallest free electric charge.) Gilbert showed that electric and magnetic forces are different. For example, a lodestone always attracts iron or other magnetic bodies. An electric object exerts its attraction only when it has been recently rubbed. On the other hand, an electric object can attract small pieces of many different substances. But magnetic forces act only between

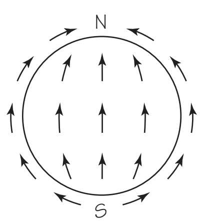


FIGURE 10.2 The Earth as a lodestone showing tiny magnets lined up at different locations on Earth.

a few types of substances. Objects are attracted to a rubbed electric object along lines directed toward one center region. But magnets always have *two* regions (poles) toward which other magnets are attracted.

Gilbert went beyond summarizing the known facts of electricity and magnets. He suggested new research problems that were pursued by others for many years. For example, he proposed that while the poles of two lodestones might either attract or repel each other, electric bodies could never exert repelling forces. However, in 1646, Sir Thomas Browne published the first observation of electric repulsion. In order to systematize such accounts, scientists introduced a new concept, *electric charge*. In the next section, you will see how this concept can be used to describe the forces between electrically charged bodies.

10.2 ELECTRIC CHARGES AND ELECTRIC FORCES

As Gilbert strongly argued, the behavior of electrified objects must be learned in the laboratory rather than by just reading about it. This section, therefore, is only a brief outline to prepare you for your own experience with the phenomena.

As discussed earlier, amber, when rubbed, acquires in a seemingly mysterious way the property of picking up small bits of grain, cork, paper, hair, etc. To some extent, all materials show this effect when rubbed, including rods made of glass or hard rubber, or strips of plastic. There are two other important basic observations:

1. When two rods of the same material are both rubbed with another material, the rods *repel* each other. Examples that were long ago found to work especially well are two glass rods rubbed with silk cloth, or two hard rubber rods rubbed with fur.
2. When two rods of *different* material are rubbed (e.g., a glass rod rubbed with silk, and a rubber rod rubbed with fur), the two rods may *attract* each other.

Electric Charges

These and thousands of similar experimentally observable facts about electrified objects can be summarized in a systematic way by adopting a very simple model. While describing a *model* for electrical attraction and repulsion, remember that this model is *not* an experimental fact which you can observe separately. It is, rather, a set of invented ideas which help to describe and summarize observations. It is easy to forget this important difference between experimentally observable facts and invented explanations. Both are needed, but they are not the same thing! The model adopted, based upon experimental evidence, consists of the concept of *charge*, along with three rules regarding charges. An object that is rubbed and given the property of attracting small bits of matter is said "to be electrically charged" or "to have an electric charge." All objects showing electrical behavior are found to have either one or the other of the two kinds of charge. The study of their behavior is known as *electrostatics*, since the charges are usually static, that is, not moving. (The study of moving charges is known as *electrodynamics*.)

The three empirical rules regarding electrostatic charges are:

1. There are only two kinds of electric charge.
2. Two objects charged alike (i.e., having the same kind of charge) repel each other.
3. Two objects charged oppositely attract each other.

When two different uncharged materials are rubbed together (e.g., the glass rod and the silk cloth), they acquire opposite kinds of charge. Benjamin Franklin, who did many experiments with electric charges, proposed a mechanical model for such phenomena. In his model, charging an object electrically involved the transfer of an "electric fluid" that was present in all matter. When two objects were rubbed together, some electric fluid from one object passed into the other. One body then had an extra amount of electric fluid and the other had a lack of that fluid. An excess of fluid produced one kind of electric charge, which Franklin called "positive." A lack of the same fluid produced the other kind of electric charge, which he called "negative."

Previously, some theorists had proposed a different, "two-fluid" model involving both a "positive fluid" and a "negative fluid." In that model, normal matter contained equal amounts of these two fluids, so that they canceled out each other's effects. When two different objects were rubbed together, a transfer of fluids occurred. One object received an excess of positive fluid, and the other received an excess of negative fluid.

There was some dispute between advocates of one-fluid and two-fluid models, but both sides agreed to speak of the two kinds of electrical charges as either *positive* (+) or *negative* (−). It was not until the late 1890s that experimental evidence gave convincing support to any model for "electric charge." Franklin thought of the electric fluid as consisting of tiny particles, and that is the present view, too. Consequently, the word "charge" is often used in the plural. For example, we usually say "electric charges transfer from one body to another."

What is amazing in electricity, and indeed in other parts of physics, is that so few concepts are needed to deal with so many different observations. For example, a third or fourth kind of charge is not needed in addition to positive and negative. Even the behavior of an *un*charged body can be understood in terms of + and − charges. Any piece of matter large enough to be visible can be considered to contain a large amount of electric charge, both positive and negative. If the positive charge is equal to the negative charge, the piece of matter will appear to have zero charge, no charge at all. The effects of the positive and negative charges simply cancel each other when they are added together or are acting together. (This is one advantage of calling the two kinds of charge positive and negative rather than, say, x and y.) The electric charge on an object usually means a slight *excess* (or net) of either positive or negative charge that happens to be on that object.

The Electric Force Law

What is the "law of force" between electric charges? In other words, how does the force depend on the *amount* of charge and on the *distance* between the charged objects?

FIGURE 10.3 Benjamin Franklin (1706–1790), American statesman, inventor, scientist, and writer, was greatly interested in the phenomenon of electricity. His famous kite experiment and invention of the lightning rod gained him wide recognition. Franklin is shown here observing the behavior of a bell whose clapper is connected to a lightning rod.

> Joseph Priestley (1773–1804), a Unitarian minister and physical scientist, was persecuted in England for his radical political ideas. One of his books was burned, and a mob looted his house because of his sympathy with the French Revolution. He moved to America, the home of Benjamin Franklin, who had stimulated Priestley's interest in science. Primarily known for his identification of oxygen as a separate element that is involved in combustion and respiration, he also experimented with electricity. In addition, Priestley can claim to be the developer of carbonated drinks (soda-pop).

The first evidence of the nature of such a force law was obtained in an indirect way. About 1775, Benjamin Franklin noted that a small cork hanging near an electrically charged metal can was strongly attracted to the can. But when he lowered the cork by a thread into the can, he found that no force was experienced by the cork no matter what its position inside the can. Franklin did not understand why the walls of the can did not attract the cork when it was inside but did when it was outside. He asked his friend Joseph Priestley to repeat the experiment.

Priestley verified Franklin's results and went on to reach a brilliant conclusion from them. He remembered from Newton's *Principia* that gravitational forces behave in a similar way. Inside a hollow planet, the net gravitational force on an object (the sum of all the forces exerted by all parts of the planet) would be exactly zero. This result also follows mathematically from the law that the gravitational force between any two individual pieces of matter is inversely proportional to the square of the distance between them

$$F \propto \frac{1}{R^2}.$$

Priestley therefore proposed that forces exerted by charges vary inversely as the square of the distance, just as do forces exerted by massive bodies. The force exerted between bodies owing to the fact that they are charged is called "electric" force, just as the force between uncharged bodies is called "gravitational" force. (Remember that all forces are known by their mechanical effects, by the push or acceleration they cause on material objects.)

Priestly had based his proposal on reasoning by analogy, that is, reasoning from a parallel, well-demonstrated case. Such reasoning alone could not *prove* that electrical forces are inversely proportional to the square of

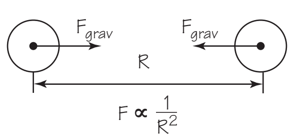


FIGURE 10.4 Two bodies under mutual gravitation.

the distance between charges. But it strongly encouraged other physicists to test Priestley's hypothesis by experiment.

The French physicist Charles Coulomb provided direct experimental evidence for the inverse-square law for electric charges suggested by Priestley. Coulomb used a *torsion balance* which he had invented (see Figure 10.5). A horizontal, balanced insulating rod is suspended by a thin silver wire. The wire twists when a force is exerted on the end of the rod, and the twisting effect can be used as a measure of the force.

Coulomb attached a charged body, a, to one end of the rod and placed another charged body, b, near it. The electrical force F_{el} exerted on a by b caused the wire to twist. By measuring the twisting effect for different separations between the centers of spheres a and b, Coulomb found that the force between spheres varied in proportion to $1/R^2$, just as Priestley had deduced

$$F_{el} \propto \frac{1}{R^2},$$

where R represents the distance between the centers of the two charges. *The electric force of repulsion for like charges, or attraction for unlike charges, varies inversely as the square of the distance between charges.*

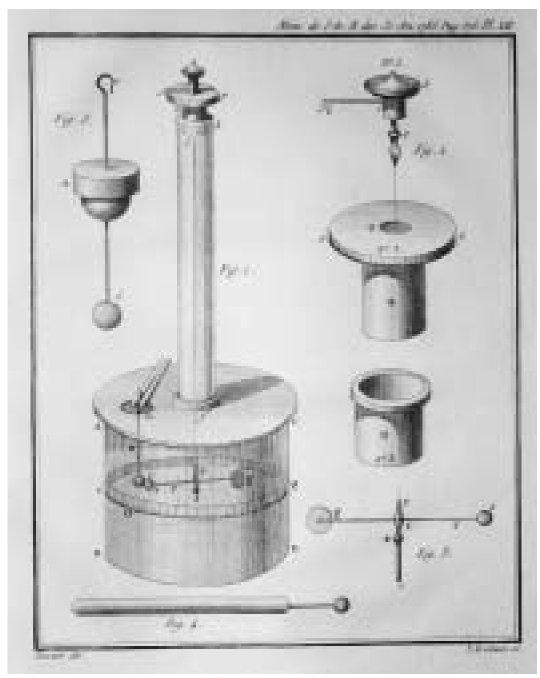


FIGURE 10.5 Coulomb's torsion balance.

Coulomb also demonstrated how the magnitude of the electric force depends on the magnitudes of the charges. There was not yet any accepted method for measuring quantitatively the amount of charge on an object. (In fact, nothing said so far would suggest how to measure the magnitude of the charge on a body.) Yet Coulomb used a clever technique based on symmetry to compare the effects of different amounts of charge. He first showed that if a charged metal sphere touches an uncharged sphere of the same size, the second sphere becomes charged also. You might say that, at the moment of contact between the objects, some of the charge from the first "flows" or is "conducted" to the second. Moreover, after contact has been made, the two spheres are found to share the original charge *equally*. (This is demonstrated by the observable fact that they exert equal forces on some third charged body.) Using this principle, Coulomb started with a given amount of charge on one sphere. He then shared this charge by contact among several other identical but uncharged spheres. Thus, he could produce charges of one-half, one-quarter, one-eighth, etc., of the

FIGURE 10.6 Charles Augustin Coulomb (1738–1806) was born into a family of high social position and grew up in an age of political unrest. He studied science and mathematics and began his career as a military engineer. While studying machines, Coulomb invented his torsion balance, with which he carried out intensive investigations on the mechanical forces caused by electrical charges. These investigations were analogous to Cavendish's work on gravitation.

original amount. In this way, Coulomb varied the charges on the two original test spheres independently and then measured the change in force between them using his torsion balance.

Coulomb found that, for example, when the charges on the two spheres are both reduced by one-half, the force between the spheres is reduced to one-quarter of its previous value. In general, he found that the magnitude of the electric force is proportional to the *product* of the charges. The symbols q_a and q_b can be used for the net charge on bodies a and b. The magnitude F_{el} of the electric force that each charge exerts on the other is proportional to $q_a \times q_b$. This may be written in symbols as

$$F_{el} \propto q_a q_b.$$

Coulomb summarized his two results in a single equation that describes the electric forces two small charged spheres A and B exert on each other

$$F_{el} = k\frac{q_a q_b}{R^2}.$$

In this equation, R represents the distance between the centers of the two charged spheres, and k is a constant whose value depends on the units of charge and of length that are used. This form of the electric force law between two electric charges is now called *Coulomb's law*. Note one striking fact about Coulomb's law: It has exactly the same form as Newton's law of universal gravitation

$$F_{grav} = G\frac{m_1 m_2}{R^2}!$$

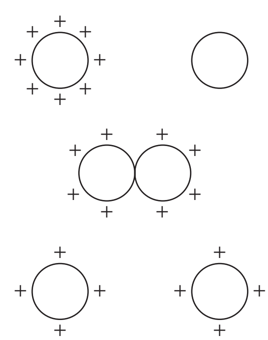


FIGURE 10.7 The equal sharing of charge between a charged and an uncharged sphere.

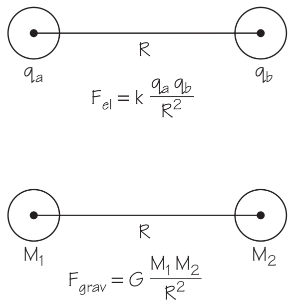


FIGURE 10.8 Magnitudes of electrical and gravitational forces between two spheres.

Here m_1 and m_2 are two masses separated by the distance R between their centers. Yet these two great laws arise from completely different sets of observations and apply to completely different kinds of phenomena. Why they should match so exactly is, to this day, a fascinating puzzle.

The Unit of Charge

Coulomb's law can be used to define a unit of charge, as long as we have defined the units of the other quantities in the equation. For example, we can assign k a value of exactly 1 and then define a unit charge so that two unit charges separated by a unit distance exert a unit force on each other. There actually is a set of units based on this choice. However, another system of electrical units, the "mksa" system, is more convenient to use. In this system, the unit of charge is derived not from electrostatics, but from the unit of current—the *ampere* (A). (See Section 10.5.) The resulting unit of charge is called (appropriately) the *coulomb* (C). It is defined as the amount of charge that flows past a point in a wire in 1 s when the current is equal to 1 A.

The ampere (A), or "amp," is a familiar unit frequently used to describe the current in electrical appliances. The effective amount of current in a common 100-W light bulb in the United States is approximately 1 A. Therefore, the amount of charge that goes through the bulb in 1 s is about 1 C. It might seem that a coulomb is a fairly small amount of charge. However, 1 C of *net* charge collected in one place is unmanageably large! In the light bulb, 1 C of negative charge moves through the filament each second. However, these negative charges are passing through a (more or less) stationary arrangement of *positive* charges in the filament. Thus, the *net* charge on the filament is zero.

Taking the coulomb (1 C) as the unit of charge, you can find the constant k in Coulomb's law experimentally. Simply measure the force between

FIGURE 10.9 A typical bolt of lightning represents about 40,000 amperes (on average) and transfers about 50 coulombs of charge between the cloud and the ground.

known charges separated by a known distance. The value of *k* turns out to equal about nine billion newton-meters squared per coulomb squared, or in symbols

$$k = 9 \times 10^9 \text{ N m}^2/\text{C}^2.$$

In view of this value of *k*, two objects, each with a *net* charge of 1 C, separated by a distance of 1 m, would exert forces on each other of nine billion N. (See if you can verify this from Coulomb's law.) This electric force is roughly as large as the gravitational force of one million tons! We can never directly observe such large electric forces in the laboratory because we cannot actually collect so much net charge (just 1 C) in one place. Nor can we exert enough force to bring two such charges so close together. The mutual repulsion of like charges is so strong that it is difficult to keep a charge of more than one-thousandth of a coulomb on an object of ordinary size. If you rub a pocket comb on your sleeve enough to produce a spark when the comb is brought near a conductor (such as a sink faucet), the net charge on the comb will be far less than one-millionth of a coulomb. Lightning discharges usually take place when a cloud has accumulated a net charge of a few hundred coulombs distributed over its very large volume.

Electrostatic Induction

As noted, and as you have probably observed, an electrically charged object can often attract small pieces of paper. But the paper itself has no net charge; it exerts no force on other pieces of paper. At first sight then, its attraction to the charged object might seem to contradict Coulomb's law. After all, the force ought to be zero if either q_a or q_b is zero.

To explain the observed attraction, recall that uncharged objects contain equal amounts of positive and negative electric charges. When a charged body is brought near a neutral object, it may rearrange the positions of some of the charges in the neutral object. The negatively charged comb does this when held near a piece of paper. Some of the negative charges in the paper shift away from the comb, leaving a corresponding amount of positive charge near the comb. The paper still has no *net* electric charge. But some of the positive charges are slightly *closer* to the comb than the corresponding negative charges are. So the attraction to the comb is greater than the repulsion. (Remember that the force gets weaker with the square of the distance, according to Coulomb's law. The force would be only one-fourth as large if the distance were twice as large.) In short, there is a net

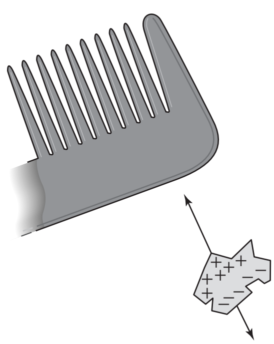


FIGURE 10.10 Electrostatic induction in neutral paper near a charged comb.

attraction of the charged body for the neutral object. This explains the old observation of the effect rubbed amber had on bits of grain and the like.

To put the observation another way: A charged body *induces* a shift of charge in or on the nearby neutral body. Thus, the rearrangement of electric charges inside or on the surface of a neutral body caused by the *influence* of a nearby charged object is called *electrostatic induction*. In Chapter 12, you will see how the theory of electrostatic induction played an important role in the development of the theory of light as an electromagnetic wave.

10.3 FORCES AND FIELDS

Gilbert described the action of the lodestone by saying it had a "sphere of influence" surrounding it. By this he meant that any other magnetic body coming inside this "sphere" would be attracted. In addition, the strength of the attractive force would be greater at places closer to the lodestone. In modern language, we should say that the lodestone is surrounded by a *magnetic field*. We can trace the magnetic field, for instance, of a bar magnet, by placing many small bits of iron fillings in the vicinity of a bar magnet that is on a table or other surface.

The world "field" is used in many different ways. Here, some familiar kinds of fields will be discussed, and then the idea of physical fields as used in science will be gradually developed. This exercise should remind you that most terms in physics are really adaptations of commonly used words, but with important changes. Velocity, acceleration, force, energy, and work are such examples, too.

One ordinary use of the concept of a field is illustrated by the "playing field" in various sports. The football field, for example, is a place where teams compete according to rules that confine the important action to the area of the field. "Field" in this case means a region of interaction.

In international politics, people speak of "spheres" or "fields" of influence. A field of political influence is also a region of interaction. But unlike a playing field, it has no sharp boundary line. A country usually has greater influence on some countries and less influence on others. So in the political sense, "field" refers also to an *amount* of influence, more in some places and less in others. Moreover, the field has a *source*, that is, the country that exerts the influence.

There are similarities here to the concept of field as used in physics. But there is also an important difference. To define a field in physics, it must

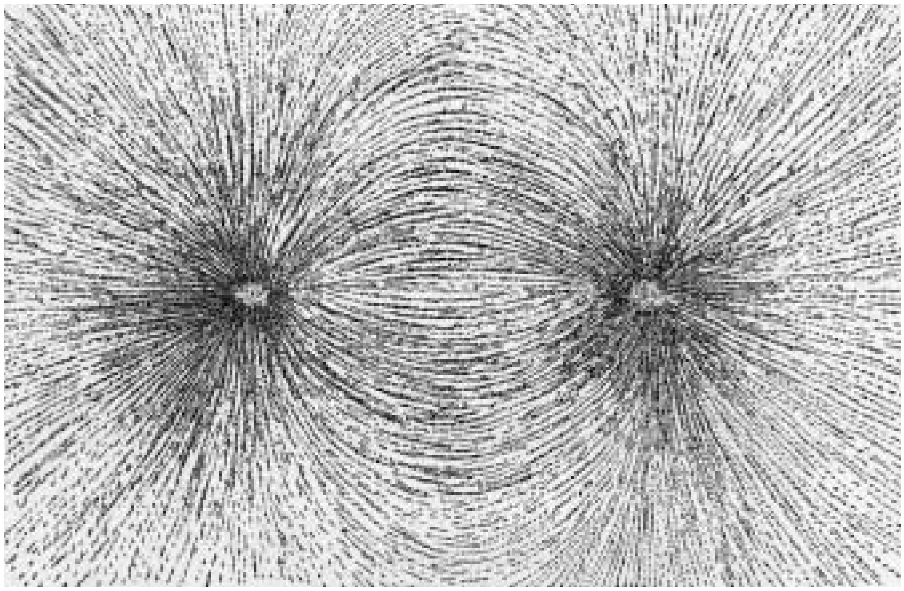


FIGURE 10.11 Iron fillings on a surface above a bar magnet align to show magnetic field lines.

be possible to assign a *numerical value* of field strength to every point in the field. This part of the field idea will become clearer if you consider some situations that are more directly related to the study of physics. First think about these situations in everyday language, then in terms of physics:

(a) You are walking along the sidewalk toward a street lamp; you observe that the brightness of the light is increasing.
(b) You are standing on the sidewalk as an automobile passes by with its horn blaring; you observe that the sound gets louder and then softer.

You can also describe these experiences in terms of fields:

(a) The street lamp is surrounded by a field of illumination. The closer you move to the lamp, the stronger is the field of illumination as registered on your eye or on a light meter (photometer) you might be carrying. For every point in the space around the street lamp, you can assign a number that represents the strength of the field of illumination at that place.
(b) The automobile horn is surrounded by a sound field. You are standing still in your frame of reference (the sidewalk). A pattern of field

values goes past you with the same speed as the car. You can think of the sound field as steady but moving with the horn. At any instance, you could assign a number to each point in the field to represent the intensity of sound. At first the sound is faintly heard as the weakest part of the field reaches you. Then the more intense parts of the field go by, and the sound seems louder. Finally, the loudness diminishes as the sound field and its source (the horn) move away.

Notice that each of the above fields is produced by a single course. In (a) the source is a stationary street lamp; in (b) it is a moving horn. In both cases the field strength gradually increases as your distance from the source decreases. One numerical value is associated with each point in the field.

So far, our examples have been simple *scalar* fields. No direction has been involved in the value of the field at each point. Figure 10.12 shows maps of two fields for the layer of air over North America on two consecutive days. There is a very important difference between the field mapped on the left and that mapped on the right. The *air pressure field* (on the left) is a scalar field; the *wind velocity field* (on the right) is a vector field. For each point in the pressure field, a single number (a scalar quantity) gives the value of the field at that point. But for each point in the wind velocity field, the value of the field is given by both a numerical value (magnitude) and a *direction*, that is, by a *vector*.

> Note that meteorologists have a convention for representing vectors different from the one we have been using. What are the advantages and disadvantages of each?

These field maps can help in more or less accurately predicting what conditions might prevail in the field on the next day. Also, by superimposing the maps for pressure and wind velocity, you can discover how these two kinds of fields are related to each other.

Physicists actually use the term "field" in three different senses:

1. the value of the field *at a point* in space;
2. the set or collection of all values everywhere in the space where that field exists;
3. the region of space in which the field has values other than zero.

In reading the rest of this chapter, you will not find it difficult to decide which meaning applies each time the term is used.

The Gravitational Force Field

Before returning to electricity and magnetism, let us illustrate a bit further the idea of a field. A good example is the gravitational force field of the Earth. The force exerted by the Earth on any object above its surface acts

PRESSURE AND VELOCITY FIELDS

These maps, adapted from those of the U.S. Weather Bureau, depict two fields, air pressure at the Earth's surface and high-altitude wind velocity, for two successive days. Locations at which the pressure is the same are connected by lines. The set of such pressure "contours" represents the overall field pattern. The wind velocity at a location is indicated by a line (showing direction) and (not visible here) feather lines—one for every 10 mi/hr. (The wind velocity over the tip of Florida, for example, is a little to the east of due north and is approximately 30 mi/hr.)

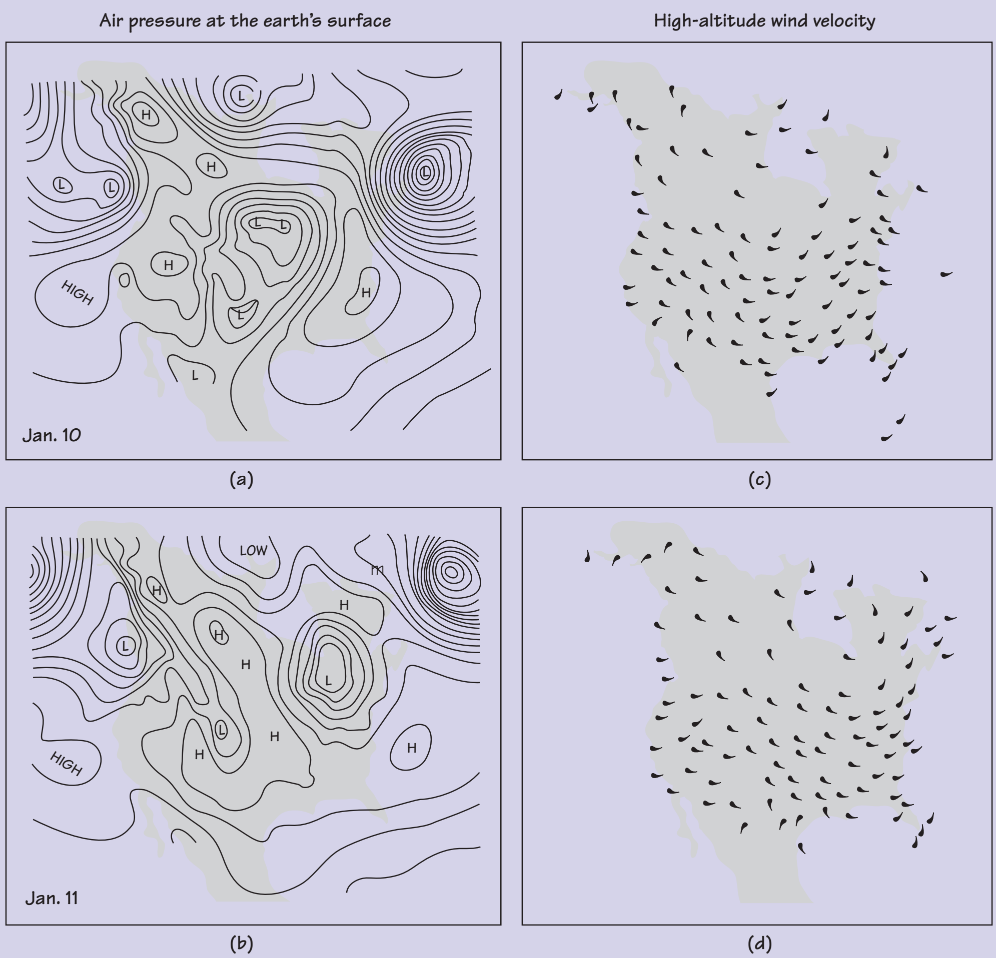


FIGURE 10.12 Weather Bureau maps of air pressure fields and wind velocity fields on two consecutive days.

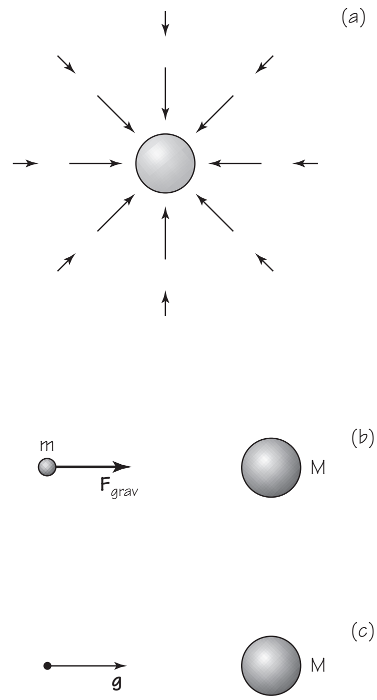


FIGURE 10.13 Gravitational force field.

in a direction toward the center of the Earth. So the field of force of gravitational attraction is a *vector* field, which can be represented by arrows pointing toward the center of the Earth. In Figure 10.13 a few such arrows are shown, some near, some far from the Earth. The strength, or numerical magnitude, of the Earth's gravitational force field at any chosen point depends on the distance of the point from the center of the Earth. This follows from Newton's theory, which states that the magnitude of the gravitational attraction is inversely proportional to the square of the distance R:

$$F_{\mathrm{grav}} = G\,\frac{Mm}{R^2},$$

where M is the mass of the Earth, m is the mass of the test body, R is the distance between the centers of Earth and the test body, and G is the universal gravitational constant.

In this equation, F_{grav} also depends on the mass of the test body. It would be more convenient to define a field that depends only on the properties of the source, whatever the mass of the test body. Then you could think of that field as existing in space and having a definite magnitude and direction at every point. The mass of the test body would not matter. In fact, it would not matter whether there were any test body present at all. As it happens, such a field is easy to define. By slightly rearranging the equation for Newton's law of gravitation, you can write

$$F_{\text{grav}} = m\frac{GM}{R^2}.$$

Then define the gravitational field strength $\mathbf{g}$ around a spherical body of mass M as having a magnitude GM/R^2 and a direction the same as the direction of $\mathbf{F}_{\text{grav}}$, so that

$$\mathbf{F}_{\text{grav}} = m\mathbf{g},$$

where the magnitude of $\mathbf{g}$ is GM/R^2. Thus, note that the magnitude of $\mathbf{g}$ at a point in space is determined by the source mass M and the distance R from the source, and does *not* depend on the mass of any test object.

The total or net gravitational force at a point in space is usually determined by more than one source. For example, the Moon is acted on by the Sun as well as by the Earth and to a smaller extent by the other planets. In order to define the field resulting from any configuration of massive bodies, take $\mathbf{F}_{\text{grav}}$ to be the *net* gravitational force due to *all* sources. Then *define* g in such a way that you can still write the simple relationship $\mathbf{F}_{\text{grav}} = m\mathbf{g}$; that is, define $\mathbf{g}$ by the equation

$$\mathbf{g} = \frac{\mathbf{F}_{\text{grav}}}{m}.$$

Thus, the gravitational field strength at any point is the *ratio* of the net gravitational force $\mathbf{F}_{\text{grav}}$ acting on a test body at that point to the mass m of the test body. The direction of the vector $\mathbf{g}$ is the same as that of $\mathbf{F}_{\text{grav}}$.

Electric Fields

The strength of any force field can be defined in a similar way. According to Coulomb's law, the electric force exerted by one relatively small charged body on another depends on the product of the *charges* of the two bodies. Consider a charge q placed at any point in the electric field set up by an-

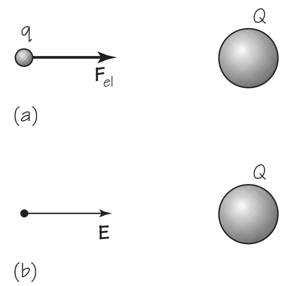


FIGURE 10.14 Electric force of charge Q on charge q (a), and the corresponding field (b).

other charge Q. Coulomb's law, describing the force F_{el} experienced by q, can be written as

$$F_{el} = K\frac{Qq}{R^2},$$

or

$$F_{el} = q\left(\frac{kQ}{R^2}\right).$$

As in the discussion of the gravitational field, the expression for force here is divided into two parts. One part, kQ/R^2, depends only on the charge Q of the source and distance R from it. This part can be called "the electric field strength owing to Q." The second part, q, is a property of the body being acted on. Thus, the vector electric field strength $\mathbf{E}$ owing to charge Q is *defined* as having magnitude kQ/R^2 and the same direction as $\mathbf{F}_{el}$. The electric force is then the product of the test charge and the electric field strength

$$\mathbf{F} = q\mathbf{E}$$

and

$$\mathbf{E} = \frac{\mathbf{F}}{q}.$$

> Recall that $\mathbf{F}_{el}$ is *called* an "electric" force because it is caused by the presence of charges. But, as with all forces, we know it exists and can measure it only by its mechanical effects on bodies.

The last equation *defines* $\mathbf{E}$ for an electric force field. Thus, the electric field strength $\mathbf{E}$ at a point in space is the *ratio* of the net electric force $\mathbf{F}_{el}$ acting on a test charge at that point to the magnitude q of the test charge. (Note that $\mathbf{E}$ is quite analogous to $\mathbf{g}$ defined earlier, but for a different field.) This definition applies whether the electric field results from a single point charge or from a complicated distribution of charges. This, true for all fields we shall encounter, is a "superposition principle." Fields set up by many sources of the same sort superpose, forming a single net field. The vector specifying the magnitude of the net field at any point is simply the vector sum of the values of the fields due to each individual source. (Once more, one marvels at the simplicity of nature and the frugality of concepts needed to describe it.)

So far, we have passed over a complication not encountered in dealing with gravitation. There are *two* kinds of electric charge, positive (+) and negative (−). The forces they experience when placed in the same electric field are opposite in direction. By agreement, scientists define the direction of the vector **E** as the direction of the force exerted by the field on a *positive* test charge. Given the direction and magnitude of the field vector at a point, then by definition the force vector $\mathbf{F}_{el}$ acting on a charge q is $\mathbf{F}_{el} = q\mathbf{E}$. A positive charge, say +0.00001 C, placed at this point will experience a force $\mathbf{F}_{el}$ in the same direction as **E** at that point. A negative charge, say −0.00001 C, will experience a force of the same magnitude, but in the *opposite* direction. Changing the sign of q from + to − automatically changes the direction of $\mathbf{F}_{el}$ to the opposite direction.

10.4 ELECTRIC CURRENTS

Touching a charged object to one end of a metal chain will cause the entire chain to become charged. The obvious explanation is that the charges move through and spread over it. Electric charges move easily through some materials, called *conductors*. Metal conductors were most commonly used by the early experimenters, but salt solutions and very hot gases also conduct charge easily. Other materials, such as glass and dry fibers, conduct charge hardly at all. Such materials are called nonconductors or *insulators*. Dry air is a fairly good insulator. (Damp air is not; you may have difficulty keeping charges on objects in electrostatic experiments on a humid day.) If the charge is great enough, however, even dry air will suddenly become a conductor, allowing a large amount of charge to shift through it. The heat and light caused by the sudden rush of charge produces a "spark." Sparks were the first obvious evidence of moving charges. Until late in the eighteenth century, a significant flow of charge, that is, an *electric current*, could be produced only by discharging a highly charged object. In studying electric currents, Benjamin Franklin believed the moving charges to be positive. Because of this, he defined the direction of flow of an electric current to be the direction of flow of positive charges. Today, we know that the moving charges in a current can be positive or negative or both. In most wires, the flowing charges are negative electrons. However, ever since Franklin's early work, *the direction of flow of an electric current is defined as the direction of flow of positive charges, regardless of the actual sign of the moving charges*. This is acceptable because the flow of negative charges in one direction is electrically equivalent to the flow of positive charges in the other direction.

In 1800, Alessandro Volta discovered a much better way of producing electric currents than using short-lived discharge devices. Volta's method involved two different metals, each held with an insulating handle. When put into contact and then separated, one metal took on a positive charge and the other a negative charge. Volta reasoned that a much larger charge could be produced by stacking up several pieces of metal in alternate layers. This idea led him to undertake a series of experiments that produced an amazing result, as reported in a letter to the Royal Society in England in March of 1800:

> Yes! the apparatus of which I speak, and which will doubtless astonish you, is only an assemblage of a number of good conductors of different sorts arranged in a certain way. 30, 40, 60 pieces or more of copper, or better of silver, each in contact with a piece

FIGURE 10.15 Alessandro Volta (1745–1827) was given his title of Baron by Napoleon in honor of his electrical experiments. He was Professor of Physics at the University of Pavia, Italy. Volta showed that the electric effects previously observed by Luigi Galvani, in experiments with frog legs, were due to the metals and not to any special kind of "animal electricity."

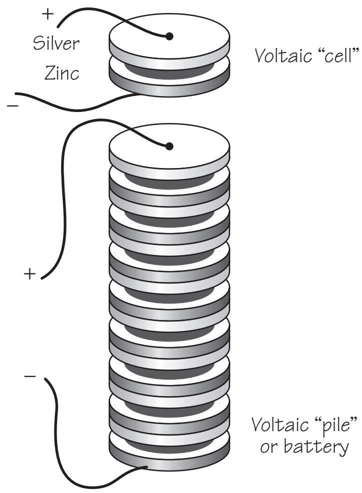


FIGURE 10.16 Voltaic cell and battery.

> of tin, or what is much better, of zinc, and an equal number of layers of water or some other liquid which is a better conductor than pure water, such as salt water or lye and so forth, or pieces of cardboard or of leather, etc., well soaked with these liquids.

Volta piled these metals in pairs, called "cells," in a vertical arrangement known as a "pile." Volta showed that one end, or "terminal," of the pile was charged positive, and the other charged negative. He then attached wires to the first and last disks of his apparatus, which he called a "battery." Through these wires, he obtained electricity with exactly the same effects as the electricity produced by rubbing amber, or by friction in electrostatic machines.

Most important of all, if the ends of the wires from Volta's battery were connected together, or attached to a conducting object, the battery produced a more or less *steady* electric current through the wires for a long period of time. This arrangement is known today as a *circuit*. The current, which flows through the wires of the circuit from the positive side of the battery to the negative side (by the earlier definition) is known as a *direct current*, or DC current. (A current that alternates in direction is known as an *alternating current*, or AC current. Most household circuits around the world provide AC current.) In addition, unlike the older charge devices, Volta's battery did not have to be charged from the outside after each use. Now the properties of electric currents as well as of static electric charges could be studied in a controlled manner. (Far better batteries have been produced as well. But one may say that Volta's invention started the series of inventions of electrical devices that have so greatly changed civilization.)

10.5 ELECTRIC POTENTIAL DIFFERENCE

Sparks and heat are produced when the terminals of an electric battery are connected. These phenomena show that energy from the battery is being transformed into light, sound, and heat energy. The battery itself converts chemical energy to electrical energy. This, in turn, is changed into other forms of energy (such as heat) in the conducting path between the terminals. In order to understand electric currents and how they can be used to transport energy, a new concept, which has the common name *voltage*, is needed.

In Chapter 5 we defined a *change in potential energy* as equal to the work required to move an object without friction from one position to another. For example, a book's gravitational potential energy is greater when the book is on a shelf than when it is on the floor. The increase in potential energy is equal to the work done in raising the book from floor to shelf. This difference in potential energy depends on three factors: the mass m of the book, the magnitude of the gravitational field strength g, and the difference in height d between the floor and the shelf.

Similarly, the *electrical* potential energy changes when work is done in moving an electric charge from one point to another in an electric field. Again, this change of potential energy $\Delta(PE)$ can be directly measured by the work that is done. The magnitude of this change in potential energy, of course, depends on the magnitude of the test charge q. Dividing $\Delta(PE)$ by q gives a quantity that does not depend on how large q is. Rather, it depends only on the intensity of the electric field and on the location of the beginning and end points. The new quantity is called *electric potential difference*. Electric potential difference is defined as *the ratio of the change in electrical potential energy* $\Delta(PE)$ *of a charge* q *to the magnitude of the charge*. In symbols

$$V = \frac{\Delta(PE)}{q}.$$

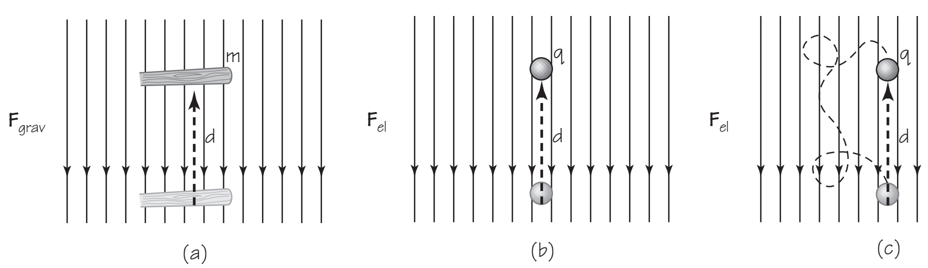


FIGURE 10.17 Gravitational and electrical potentials. The electric potential difference is the same between two points, regardless of the path.

As is true for gravitational potential energy, there is no absolute zero level of electric potential energy. The *difference* in potential energy is the significant quantity. The symbol v is used both for "potential difference" as in the equation above, and as an abbreviation for volt, the unit of potential difference (as in 1 V = 1 J/C).

The units of electric potential difference are those of energy divided by charge, or joules per coulomb. The term used as the abbreviation for joules per coulomb is *volt* (V). The electrical potential difference (or *voltage*) between two points is 1 V if 1 J of work is done in moving 1 C of charge from one point to the other

$$1 \text{ volt} = 1 \text{ joule/coulomb} \equiv 1 \text{ J/C}.$$

The potential difference between two points in a steady electric field depends on the location of the points. It does *not* depend on the *path* followed by the test charge. Whether the path is short or long, direct or roundabout, the same work is done per unit charge. Similarly, a hiker does the same work against the gravitational field per kilogram of mass in the pack he or she is carrying, whether climbing straight up or spiraling up along the slopes. Thus, the electrical potential difference between two points in a field is similar to the difference in gravitational potential energy between two points.

A simple case will help you to see the great importance of this definition of potential difference. Calculate the potential difference between two points in a uniform electric field of magnitude E produced by oppositely charged parallel plates. Work must be done in moving a positive charge q from one point to the other directly against the direction of the electric force. The amount of work required is the product of the force F_{el} exerted on the charge (where $F_{el} = qE$), and the distance d through which the charge is moved in the same direction. Thus,

$$\Delta(PE) = qEd.$$

Substituting this expression for $\Delta(PE)$ in the definition of electric potential difference gives, for the simple case of a uniform field,

$$V = \frac{\Delta(PE)}{q} = \frac{qEd}{q} = Ed.$$

In practice it is easier to measure electric potential difference V (with a voltmeter) than to measure electric field strength E. The relationship just

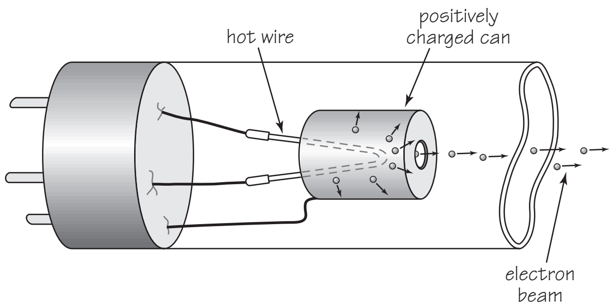


FIGURE 10.18 Electrically charged particles (electrons) are accelerated in an "electron gun" as they cross the potential difference between a hot wire (filament) and a "can" in an evacuated glass tube.

given is often useful in the form $E = V/d$, which can be used to find the intensity of a uniform electric field.

Electric potential energy, like gravitational potential energy, can be converted into kinetic energy. A charged particle placed in an electric field, but free of other forces, will accelerate. In doing so, it will increase its kinetic energy at the expense of electric potential energy. (In other words, the electric force on the charge acts in such a way as to push it toward a region of lower potential energy.) A charge q "falling" through a potential difference V increases its kinetic energy by qV if nothing is lost by friction.

$$\Delta(KE) = qV.$$

The amount of *increase* in kinetic energy is equal to the *decrease* in potential energy. So the sum of the two at any moment remains constant. This is just one particular case of the general principle of energy conservation, even though only electric forces are acting.

The conversion of electric potential energy to kinetic energy is used in *electron accelerators* (a common example is a television picture tube or some computer monitors). An electron accelerator usually begins with an electron "gun." The "gun" has two basic parts: a wire and a metal can in an evacuated glass tube. The wire is heated red-hot, causing electrons to escape from its surface. The nearby can is charged positively, producing an electric field between the hot wire and the can. The electric field accelerates the electrons through the vacuum toward the can. Many electrons stick to the can, but some go shooting through a hole in one end of it. The stream of electrons emerging from the hole can be further accelerated or focused by additional charged cans.

Such a beam of charged particles has a wide range of uses both in technology and in research. For example, a beam of electrons can make a fluorescent screen glow, as in a television picture tube or an electron microscope, or they can produce X rays for medical purposes or research. If a beam of heavier charged particles is used, they can break atoms apart for the study of their composition. When moving through a potential difference of 1 V, an electron with a charge of 1.6×10^{-19} C increases its kinetic energy by 1.6×10^{-19} J, in accord with the equation

$$\Delta(KE) = qV.$$

This amount of energy is called an *electron volt*, which is abbreviated eV. Multiples are 1 keV (= 1000 eV), 1 MeV (= 10^6 eV), and 1 GeV (= 10^9 eV). Energies of particles in accelerators are commonly expressed in such multiples. In a television tube, the electrons in the beam are accelerated across an electric potential difference of about 25,000 V. Thus,

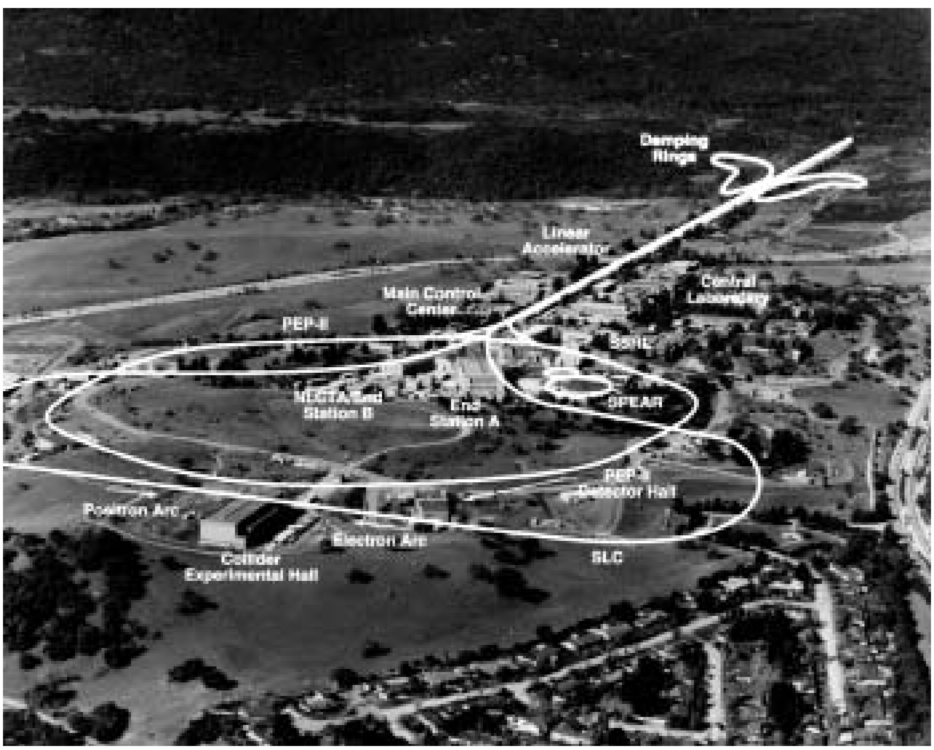


FIGURE 10.19 Stanford Linear Accelerator, with underground features highlighted.

each electron has an energy of about 25 keV. Large accelerators now operating can accelerate charged particles to kinetic energies of about 800 GeV.

10.6 ELECTRIC POTENTIAL DIFFERENCE AND CURRENT

The acceleration of an electron by an electric field in a vacuum is the simplest example of a potential difference affecting a charged particle. A more familiar example is electric current in a metal wire. In this arrangement, the two ends of the wire are attached to the two terminals of a battery. Chemical changes inside a battery produce an electric field that continually drives charges to the terminals, making one charged negatively, leaving the other charged positively. The "voltage" of the battery tells how much energy per unit charge is available when the charges move in any external path from one terminal to the other, for example, along a wire.

> In metallic conductors, the moving charge is the negative electron, with the positive atom fixed. But all effects are the same as if positive charge were moving in the opposite direction. By an old convention, the latter is the direction usually chosen to describe the direction of current.

Electrons in a metal do not move freely as they do in an evacuated tube, but continually interact with the metal atoms. If the electrons were really completely free to move, a constant voltage would make them *accelerate* so that the current would increase with time. This does not happen. A simple relation between current and voltage, first found by Georg Simon Ohm, is at least approximately valid for most metallic conductors: *The total current* I *in a conductor is proportional to the potential difference* V *applied between the two ends of the conductor*. Using the symbol I for the current, V for the potential difference, and $\propto$ for proportionality, we may write

$$V \propto I$$

or

$$V = \text{constant} \times I.$$

This simple relation is called *Ohm's law*. It is usually written in the form

$$V = IR,$$

where R is a constant called the *resistance* of the conducting path. It is measured in units of ohm, symbol Ω (Greek letter omega). Ohm's law may be stated a different way: *The resistance* R *is constant for different values of voltage and current*.

FIGURE 10.20 Georg Simon Ohm (1789–1854).

Resistance depends on the material and dimensions of the path, such as the length and diameter of a wire. For example, a thin tungsten wire has a much larger resistance than the same length of a fat copper wire. But Ohm's law assumes that the resistance R of a given conducting path does not depend on the current or the voltage. In fact, resistance is not strictly constant for any conducting path; it varies with changes in temperature, for example. But Ohm's law applies closely enough for practical technical work, though it does not have the general validity of the law of universal gravitation or Coulomb's law.

10.7 ELECTRIC POTENTIAL DIFFERENCE AND POWER

Suppose a charge could move freely from one terminal to the other in an evacuated tube. The work done on the charge would then simply increase the kinetic energy of the charge. However, a charge moving through some material such as a wire transfers energy to the material by colliding with atoms. Thus, at least some of the work goes into heat energy owing to the increased vibration of the atoms. A good example of this process is a flashlight bulb. A battery forces charges through the filament wire in the bulb. The electric energy carried by the charges is converted to heat energy in the filament. The hot filament in turn radiates energy, a small fraction of which is in the form of visible light. Recall now that "voltage" (electric potential difference) is the amount of *work* done per unit of charge transferred. So the product of voltage and current gives the amount of *work* done per unit *time*:

$$V \text{ (joules/coulomb)} \times I \text{ (coulombs/second)} = VI \text{ (joules/second)}.$$

Work done per unit time is called *power*, symbol P. The unit of electrical power, equal to 1 J/s, is called a *watt* (W). Using the definition of ampere (1 C/s) and volt (1 J/C), the equation for power P provided to an electrical circuit is

$$P \text{ (watts)} = V \text{ (volts)} \times I \text{ (amperes)}.$$

Thus, a 60-W bulb, connected to a 110-V circuit, carries 0.5 A of current.

What energy transformation does this equation imply? During their motion, the charges do work against material by colliding with atoms in the conductor. Thus the electric energy of the charges is converted into heat energy. Using the relation $P = VI$ and substituting IR for V, we obtain the important equation

$$P = IR \times I,$$

$$P = I^2R.$$

Thus, *the heat produced each second (given by* P, *the energy available per second) by a current is proportional to the square of the current*. Joule was the first to find this relationship experimentally. The discovery was part of his series of researches on conversion of different forms of energy (Chapter 6). The fact that heat production is proportional to the *square* of the current is very important in making practical use of electric energy. You will learn more about this in Chapter 11.

10.8 CURRENTS ACT ON MAGNETS

Early in the eighteenth century, reports began to appear that lightning changed the magnetization of compass needles and made magnets of knives and spoons. Some researchers believed that they had magnetized steel needles by discharging a charged object through them. These reports suggested

FIGURE 10.21 Needle-like iron oxide crystals in the magnetic field of a bar magnet. The bar magnet is under a paper on which the iron oxide crystals have been spread.

that electricity and magnetism were closely related in some way. But the casual observations were not followed up with deliberate, planned experiments that might have led to useful concepts and theories.

None of these early reports surprised the nineteenth-century Nature Philosophers in Europe. They were convinced that all phenomena observed in nature were only different effects of a single "force." Their belief in the *unity* of physical forces naturally led them to expect that electrical and magnetic forces were associated or related in some way.

The first concrete evidence of a connection between electricity and magnetism came in 1820, when the scientist Hans Christian Oersted of Copenhagen, Denmark performed an extremely important series of experiments. Oersted placed a long horizontal wire directly beneath a magnetic compass needle. The wire thus lay along the Earth's magnetic north–south line, with the magnetic needle naturally lined up parallel to the wire. When Oersted connected the wire to the terminals of a battery, the compass needle swung toward an east–west orientation, nearly perpendicular to the wire! Charge at rest does not affect a magnet. But here was evidence that charge in motion (a current) does exert an odd kind of force on a magnet.

Oersted's results were the first ever found in which a force did *not* act along a line connecting the sources of the force. (Forces between planets, between electric charges, or between magnetic poles all act along such a line.) The force exerted between the current-carrying wire and each magnetic pole of the compass needle is not along the line from the wire to the pole. In fact, for the needle to twist as it does, the force must be acting *perpendicular* to such a line. The magnetic needle is *not* attracted or repelled by the wire, but is *twisted* sideways by forces on its poles.

This was a totally new kind of effect. No wonder it had taken so long before anyone found the connection between electricity and magnetism. Closer examination revealed more clearly what was happening in this experiment. The long, straight, current-carrying wire sets up a magnetic field around it, given the symbol **B**. This field turns a small magnet so that the north–south line on the magnet is tangent to a circle whose center is at the wire and whose plane lies *perpendicular* to the wire. Thus, the current produces a *circular* magnetic field, not a centrally directed magnetic field as had been expected.

The direction of the magnetic field vector at each point is defined as *the direction of the force on the north-seeking pole of a compass needle placed at that*

HANS CHRISTIAN OERSTED

Hans Christian Oersted (1777–1851), a Danish physicist, studied the writings of the Nature Philosopher Schelling and wrote extensively on philosophical subjects himself. In an essay published in 1813, Oersted predicted that a connection between electricity and magnetism would be found. In 1820, he discovered that a magnetic field surrounds an electric current when he placed a compass over a current-carrying wire. In later years he vigorously denied the suggestion of other scientists that his discovery of electromagnetism had been accidental.

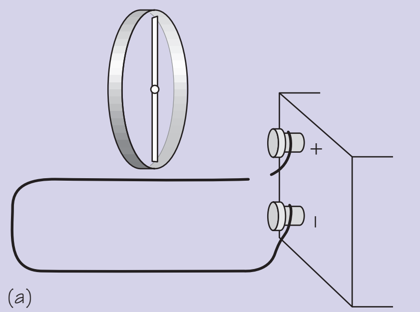


FIGURE 10.22 Oersted's experiment. (a) No current. Compass needle put across a wire points N–S. (b) Current flows. Compass needle is twisted to E–W position by magnetic force produced by current in top part of wire.

FIGURE 10.23

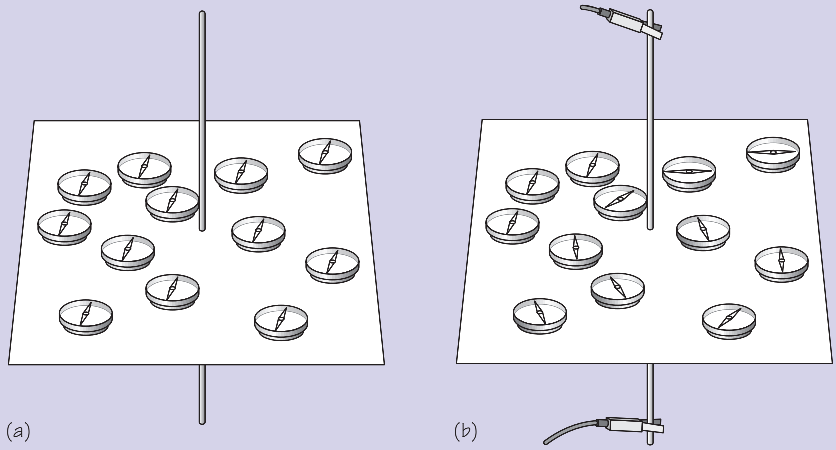


FIGURE 10.24 Representation of an array of tiny compasses on a sheet of cardboard placed perpendicular to a brass rod. When there is a strong current in the rod as in (b), the compass needles are deflected from their normal north-south line by the magnetic field set up by the current. This experiment, too, indicates that the lines of magnetic force owing to the current are circular around the rod.

point. Conversely, the force on the south-seeking pole will be in a direction exactly opposite to the field direction. A compass needle will respond to these forces on each end by turning until it points as closely as possible in the direction of the field. We noted earlier that with a bar magnet you can obtain the "shape" of the magnetic field by sprinkling tiny slivers of iron on a sheet placed over the magnet. In the same way, the magnetic field around a current-carrying wire can be visualized (see Figure 10.26). The

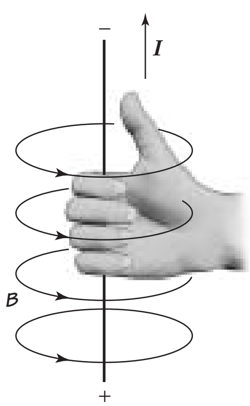


FIGURE 10.25 Remember this useful rule: If the thumb points in the direction of the flow of charge, the fingers curl in the direction of the lines of the magnetic field. The magnitude of the magnetic field is discussed in Sec. 14.13. Use the right hand for positive charge flow, left hand for negative charge flow.

slivers become magnetized and behave like tiny compass needles, indicating the direction of the field. The slivers also tend to link together end-to-end. Thus, the pattern of slivers indicates magnetic lines of force around any current-carrying conductor or bar magnet. These lines form a "picture" of the magnetic field.

You can use a similar argument to find the "shape" of a magnetic field produced by a current in a *coil* of wire, instead of a straight wire. To do this, bend the wire into a loop so that it goes through the paper in two places. The magnetic effects of the different parts of the wire on the iron slivers produce a field pattern similar to that of a bar magnet.

10.9 CURRENTS ACT ON CURRENTS

Oersted's experiment was one of those precious occasions when a discovery suddenly opens up an exciting new subject of research—a whole "ocean of ignorance" waiting to be conquered. In this case, no new equipment was needed. At once, dozens of scientists throughout Europe and America began intensive studies on the magnetic effects of electric currents. The work of André-Marie Ampère (1775–1836) stands out among all the rest. Ampère was called the "Newton of electricity" by James Clerk Maxwell, who decades later constructed a complete theory of electricity and magnetism. Ampère's work is filled with elegant mathematics. Without describing his theory in detail, we can trace some of his ideas and review some of his experiments.

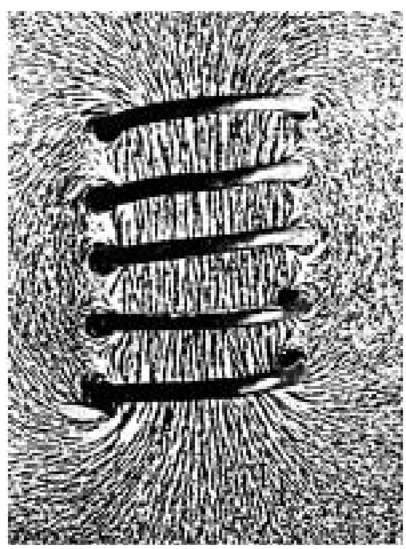


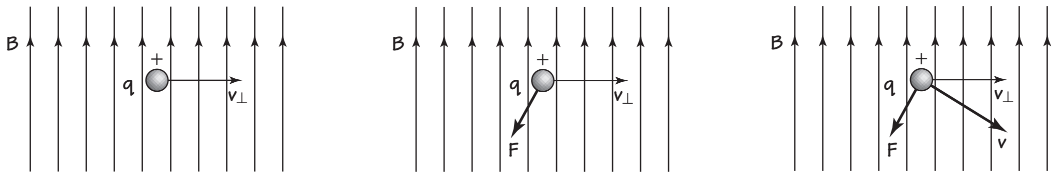
FIGURE 10.26 Iron fillings in the magnetic field produced by current in a coil of wire.

FIGURE 10.27 André-Marie Ampère (1775–1836) was born in a village near Lyons, France. There was no school in the village, and Ampère was self-taught. His father was executed during the French Revolution, and Ampère's personal life was deeply affected by his father's death. Ampère became a professor of mathematics in Paris and made important contributions to physics, mathematics, and the philosophy of science.

Ampère's thoughts raced forward as soon as he heard Oersted's news. He began with a line of thought somewhat as follows: Magnets exert forces on each other, and magnets and currents exert forces on each other. Do currents then exert forces on other currents? The answer is not necessarily yes. Arguing from symmetry is inviting, and often turns out to be right. But the conclusions to which such arguments lead are not logically or physically necessary. Ampère recognized the need to let experiment answer his question. He wrote:

> When Monsieur Oersted discovered the action which a current exercises on a magnet, one might certainly have suspected the existence of a mutual action between two circuits carrying currents; but this was not a necessary consequence; for a bar of soft iron also acts on a magnetized needle, although there is not mutual action between two bars of soft iron.

Ampère put his hunch to the test. On September 30, 1820, within a week after word of Oersted's work reached France, Ampère reported to the



(*a*) When the charge *q* moves with velocity **v** perpendicular to **B**,

(*b*) there is a force **F** as shown, directly out of plane of paper, proportional to *q*, $\mathbf{v}_\perp$, and **B**.

(*c*) If **v** is not perpendicular (⊥) to **B**, there is a smaller force, proportional to **v** instead of $\mathbf{v}_\perp$.

FIGURE 10.28 Magnetic field and force on moving charge.

French Academy of Sciences. He had indeed found that two parallel current-carrying wires exert forces on each other. They did so even though the wires showed no evidence of *net* electric charges.

Ampère made a thorough study of the forces between currents. He investigated how they depend on the distance between the wires, the relative positions of the wires, and the amount of current. In the laboratory, you can repeat these experiments and work out the "force law" between two currents. We need not go into the quantitative details here, except to note that the force between currents is easy to measure, and in turn can be used to measure how much current is flowing. In fact, the magnetic force between currents is now the quantity preferred for *defining* the unit of current. This unit is called the *ampere*, as mentioned in Section 10.2. One ampere (1 A) is defined as the amount of current in each of two long, straight, parallel wires, set 1 m apart, that causes a force of exactly 2×10^{-7} N to act on each meter of each wire.

Question: How many amps of current are flowing through the filament of a 100-W electric light bulb when connected to a 120-V outlet?

Answer: Referring to the relationship between power (in watts), voltage (in volts), and current (in amps) in Section 10.7:

$$P = VI,$$

or

$$I = \frac{P}{V},$$

$$I = \frac{100\ \text{W}}{120\ \text{V}} = 0.83\ \text{W/V} = 0.83\ \text{A}.$$

10.10 MAGNETIC FIELDS AND MOVING CHARGES

In the last two sections, the interactions of currents with magnets and with each other were discussed. The concept of *magnetic field* greatly simplifies the description of these phenomena.

As you saw in studying Coulomb's law, electrically charged bodies exert forces on each other. When the charged bodies are at rest, the forces are "electric" forces, or Coulomb forces. "Electric fields" act as the sources of these forces. But when the charged bodies are moving (as when two parallel wires carry currents), new forces in *addition* to the electric forces are present. These new forces are called "magnetic" and are caused by "magnetic fields" set up by the moving charges.

Magnetic interaction of moving charged bodies is not as simple as electric interaction. Remember the description of Oersted's experiment. The direction of the force exerted by a field set up by a current on a magnet needle is perpendicular both to the direction of the current and to the line between the magnet needle and the current. For the moment, however, it is not necessary to examine the forces on current-carrying conductors. After all, the force on a wire is believed to be caused by forces on the individual electric charges moving in it. How do such individual charges behave when moving freely in an external magnetic field? Once some simple rules have been established for the behavior of free charged particles, current in wires will be discussed further in the next chapter. There you will see how these simple rules are enough to explain the operation of electric generators and electric motors. (You will also see how these inventions have changed civilization.)

The rules summarized in the remainder of this section are best learned in the laboratory. All you need is a magnet and a device for producing a beam of charged particles, for example, the "electron gun" in an oscilloscope tube.

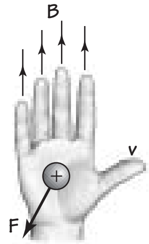


FIGURE 10.29 Remember this useful rule: If your fingers point along **B** and your thumb along v, F will be in the direction your palm would push. For positive charges use the right hand, and for negative use the left hand.

The Force on a Moving Charged Body

Suppose you have a fairly uniform magnetic field, symbol **B**, produced either by a bar magnet or by a current in a coil. How does this external field act on a moving, charged body (say, an electron)? You can find by experiment that the charge experiences a force and that the force depends on three quantities:

(1) the charge q of the body;
(2) the velocity $\mathbf{v}$ of the body; and
(3) the strength of the external field **B** through which the body is moving.

The force depends not only on the *magnitude* of the velocity, but also on its *direction*. If the body is moving in a direction *perpendicular* to the field **B**, the magnitude of the force is proportional to *both* of these quantities; that is,

$$\mathbf{F} \propto qv\mathbf{B}$$

which can also be written as

$$\mathbf{F} = kqv\mathbf{B},$$

where k is a proportionality constant that depends on the units chosen for **F**, q, v, and **B**.

But if the charge is moving in a direction *parallel* to **B**, there is no force on it, since the angle is zero between the field and the velocity vector of the charge. For all other directions of motion, the magnitude of the force is somewhere between the full value and zero. In fact, the force is proportional to the *component* of the velocity that is perpendicular to the field direction. We give this the symbol $v_{\perp}$. Therefore, a more general expression for the force is

$$\mathbf{F} \propto qv_{\perp}\mathbf{B}$$

or

$$\mathbf{F} = kqv_{\perp}\mathbf{B},$$

where k is the same constant as before. *The direction of the force is always perpendicular to the direction of the field. It is also perpendicular to the direction of motion of the charged body.*

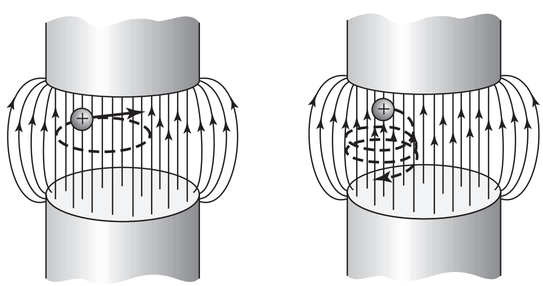
The force exerted by an external magnetic field on a moving charged particle can be used to *define* the unit of magnetic field **B**. This is done by taking the proportionality constant k as equal to one. This definition is convenient here, since we are dealing mainly with how magnetic fields act on moving charges (rather than with forces between bar magnets). So in the special case when **B** and v are *at right angles* to each other, the magnitude of the deflecting force becomes simply

$$F = qvB.$$

The Path of a Charged Body in a Magnetic Field

The force on a moving charged body in a magnetic field is always "off to the side"; that is, the force is perpendicular to the body's direction of motion at every moment. Therefore, the magnetic force does not change the *speed* of the charged body. This is analogous to the case of the gravitational force from the Sun acting on a planet in (near enough) circular orbit. Rather, in each case the central force changes the *direction* of the velocity vector, but not its magnitude. If a charged body is moving exactly perpendicular to a uniform magnetic field, there will be a constant sideways push. The body will move along a circular path, in a plane perpendicular to the direction of the magnetic field. If **B** is strong enough, the particle will be trapped in a circular orbit.

What if the charged body's velocity has some component along the direction of the field but not exactly parallel to it? The body will still be deflected into a curved path (see Figure 10.30), but the component of its motion *along* the field will continue undisturbed. So the particle will trace out a coiled (helical) path (Figure 10.30b). If the body is initially moving exactly parallel to the magnetic field (Figure 10.30a), there is no deflecting force at all, since $v_\perp$ is zero.



(a) (b)

FIGURE 10.30 Charged particle in **B** field.

Some important examples of the deflection of charged particles by magnetic fields include particle accelerators and bubble chambers. One example of "coiled" motion is found in the Van Allen radiation belts, a doughnut-shaped region encircling the Earth and extending from a few hundred kilometers to about fifty thousand kilometers above the Earth's surface. A stream of charged particles, mainly from the Sun, but also from outer space, continually sweeps past the Earth. Many of these particles are deflected into spiral paths by the magnetic field of the Earth and become "trapped" in the Earth's field. Trapped particles from the geomagnetic tail sometimes spiral their way toward the Earth's magnetic poles. When they hit the atmosphere, they excite the atoms of the gases to radiate light. This is the cause of the aurora ("northern lights" and "southern lights").

Since magnets acts on currents and currents act on currents, the forces produced by magnets and currents can be used to perform work by pushing or pulling on the currents on which they act. The harnessing of this work in useful forms had profound economic and social consequences. This behavior of magnets and currents made possible the invention of the electric motor, which turns electrical energy into mechanical work, and the electric generator, which turns mechanical work into electrical energy. Together these devices helped to open the electric age, in which we still live.

FIGURE 10.31 The glow of the aurora borealis is produced when the upper atmosphere is excited by charged particles moving through the Earth's magnetic field.

SOME QUANTITATIVE EXPRESSIONS IN THIS CHAPTER

Magnitude of gravitational force on m_1 or m_2:

$$F_{\text{grav}} = G\frac{m_1 m_2}{R^2}.$$

Magnitude of electric force on q_1 or q_2:

$$F_{\text{el}} = k_{\text{el}}\frac{q_1 q_2}{R^2}.$$

Magnitude of magnetic force on a moving charge, if v and B are perpendicular:

$$F = qvB.$$

Gravitational field strength:

$$\mathbf{g} = \frac{\mathbf{F}_{\text{grav}}}{m}.$$

Electric field strength, **E**, is defined as the electric force per unit of test charge:

$$\mathbf{E} = \frac{\mathbf{F}_{\text{e}}}{q}.$$

Ohm's law:

$$V = IR.$$

Potential difference, V, is defined as the ratio of the change in electrical potential energy of charge to the magnitude of the charge:

$$V = \frac{\Delta PE}{q}.$$

Electric power:

$$P = VI \qquad \text{or} \qquad P = I^2R.$$

SOME IMPORTANT NEW UNITS

One amp of current = one coulomb of charge per second, 1 A = 1 C/s.

1 volt = 1 joule/coulomb, 1 V = 1 J/C.

1 ohm = 1 volt/amp, 1 Ω = 1 V/A.

SOME NEW IDEAS AND CONCEPTS

AC current
ampere
charge
conductors
Coulomb's law
current
DC current
electric field
electric field strength
electrodynamics
electrostatic induction
electrostatics
insulators
lines of force
lodestone
magnetic field
Ohm's law
pole
potential difference
voltage

FURTHER READING

G. Holton and S.G. Brush, *Physics, The Human Adventure* (Piscataway, NJ: Rutgers University Press, 2001), Chapter 24.
B. Franklin, *The Autobiography of Benjamin Franklin* (New York: Dover, 1996), and many other editions.

STUDY GUIDE QUESTIONS*

10.1 Gilbert's *Magnets*

1. What are amber and loadstone?
2. What are some of the strange properties of amber and loadstone?
3. How did Gilbert make it plausible that the Earth behaves like a spherical loadstone?

* These questions are intended as an aid to study. Your instructor may ask you to work on these in groups, or individually, or as a class.

4. How does the attraction of objects by amber differ from the attraction by lodestone?

10.2 Electric Charges and Electric Forces

1. What are some important observations about static charges?
2. What are the three general rules about electric charges?
3. Why did people think that electricity involves a fluid?
4. How did Priestley discover that the electric force is proportional to $1/R^2$?
5. How did Coulomb arrive at the electric force law?
6. Compare and contrast the electric, magnetic, and gravitational forces.
7. If the distance between two charged objects is doubled, how is the electric force between them affected? How is the force affected if the charge on one of them is cut to one-quarter its former size.
8. State Coulomb's law in your own words. What is the direction of the force between two charged particles? What is the value of k?

10.3 Forces and Fields

1. What is the difference between a scalar field and a vector field?
2. Describe how you can find by experiment the magnitude and directions of:
   (a) the gravitational field at a certain point in space;
   (b) the electric field at a certain point in space.
3. On a sheet of paper draw a circle to represent a ball of charge. Label it a negative charge, i.e., $-q$. Then choose three points on the paper and draw the direction of the electric field at each point. Finally, draw the direction of the force vector on a negative charge at each point.
4. A negative test charge is placed in an electric field where the electric field vector is pointing downward. What is the direction of the force on the test charge?
5. What is the electric field at a point if a test particle of 3×10^{-5} C experiences a force of 10^{-2} N upward?

10.4 Electric Currents

1. Describe Volta's pile and how it operated.
2. What were some of the advantages of the pile?
3. What is the difference between AC and DC current?

10.5 Electric Potential Difference

1. How is the electric potential difference between two points defined?
2. Does the potential difference between two points depend on:
   (a) the path followed as a charge moves from one point to the other?
   (b) the magnitude of the charge that moves?
3. How is the unit of electron-volt (eV) defined?
4. What are eV units used in some situations instead of Joules? What are some situations in which they are used?

10.6 Electric Potential Difference and Current

1. How does the current in a conductor change if the potential difference between the ends of the conductor is doubled?
2. How would you test whether Ohm's law applies to a given piece of wire?

10.7 Electric Potential Difference and Power

1. What happens to the electrical energy used to move a charge through a conducting material?
2. How does the energy per second converted to heat in a conductor change if the current in the conductor is doubled?
3. Show that the kilowatt-hour is a measure of energy and not of power.

10.8 Currents Act on Magnets

1. Describe, with drawings, Oersted's discovery. Why is it important to this day?
2. Under what conditions can electric charges affect magnets?
3. What was surprising about the force a current exerted on a magnet?
4. What are the shape and direction of the magnetic field produced near a straight wire carrying a steady current?

10.9 Currents Act on Currents

1. What was Ampère's hunch?

10.10 Magnetic Fields and Moving Charges

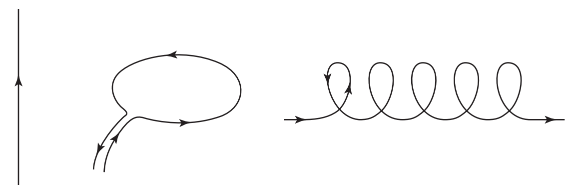
1. Which of the following affect the magnitude of the deflecting force on a moving charged particle?
   (a) The component of the velocity parallel to the magnetic field?
   (b) The component of the velocity perpendicular to the magnetic field?
   (c) The magnetic find itself?
   (d) The magnitude of the charge?
   (e) the sign of the charge?
2. What are differences between deflecting forces on a charged object due to:
   (a) gravity?
   (b) an electric field?
   (c) a magnetic field?

DISCOVERY QUESTIONS*

1. What is an electromagnet and how does it work?
2. A battery is used to light a flashlight bulb. Trace as many of the energy conversions as you can in this circuit.

* These questions go beyond the text itself.

3. What are the magnetic fields around the wires carrying steady currents in the figures below?



(a) (b) (c)

4. Show that for a charge Q moving a distance D under the influence of an electric field E, its kinetic energy will be $KE = QED$.

Quantitative

In the following, take the value of k_e in Coulomb's law to be about nine billion newton-meters squared per coulomb squared, or $k_e = 9 \times 10^9$ N m^2/C^2. The value of the gravitational constant G is 6.67×10^{-11} N m^2/kg^2.

1. If you compare the gravitational force between two 1-kg masses with the electric force between two 1-C charges at the same distance, which force is larger, and by how much?
2. What is the electric force between two positive charges of 1 C placed 1 m from each other?
3. How does the force in Question 2 compare with the gravitational force between two 1-kg masses placed 1 m apart?
4. What is the electric field at a point where a test charge of 3×10^{-5} C experiences a force of 10^{-2} N upward?
5. The electric charge on an electron is -1.6×10^{-19} C. How many electrons would be required to make -1 C of charge?
6. An electron crosses a potential of 10 V. What is the change in its kinetic energy, in eV?
7. An alpha particle (helium nucleus of charge $+2e$) crosses the same potential of 10 V. What is the change in its kinetic energy, in eV? How many joules is this?
8. Find out the electric power ratings of some common household devices that you would normally use. During the course of 1 day, note which devices you use and for approximately how long. Include light bulbs, TV, computer, a hair dryer, electric water heater, washer and dryer, etc. Obtain your total energy consumption for that day. How many hours could your total energy consumption keep a 100-W light bulb burning?
9. A household consumed 5500 kilowatt-hours in 1 month.
   (a) How many joules of energy is this?
   (b) How many hours could this amount of energy keep a 100-W light bulb burning?
10. What is the power used by a circuit in which 3 A flow across a 240-V supply? What is the resistance of this circuit?