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CHAPTER 15

Quantum Mechanics

15.1 THE QUANTUM

The discovery of the quantum of energy in the early years of the twentieth century provided an explanation of the photoelectric effect and it enabled the success of Bohr's quantum model of the atom. This model and the other successes of that time contributed to what is known as "quantum theory."

Nevertheless, the existence of the energy quantum, whether in light or in atoms, posed a serious problem for physics, since it was incompatible with both Newton's mechanics and Maxwell's electromagnetic wave theory. In these theories energy is always continuous and infinitely divisible. But these "classical" theories were constructed on the basis of events occurring in the visible, human-scale world, ranging from planets to microscopic objects. Perhaps, then, we should not be surprised that nature might behave differently when we enter into regions far removed from everyday experience, such as the interior of atoms or the sub-microscopic structure of minute quantities of light. And when we are surprised, it only impresses upon us even more how intricate and exquisite nature really is.

By the mid-1920s scientists realized that the quantum theory was inadequate and that a new theory was needed to encompass the quantum world at the subatomic level, a new *quantum mechanics* in which the quantum is built into the foundations of physics from the beginning. An important clue to the new mechanics came from the further study of particles and waves.

15.2 THE PARTICLE-LIKE BEHAVIOR OF LIGHT

Einstein's hypothesis of light quanta created a dilemma for physicists. While Einstein's work indicated that light behaves like particles in such experiments as the photoelectric effect, light clearly behaved like waves in Young's important double-slit experiment. As you may recall from Chapter 8, when a beam of light shines on two narrow slits near each other, the light emerging from the slits interferes and forms on a screen the alternating bright and dark bands that are characteristic of the interference of waves. Particles cannot form this pattern. Moreover, Maxwell's electromagnetic theory accounted for electromagnetic radiation as a wave phenomenon, and this theory was supported by Young's experiment and many other experiments.

On the other hand, Einstein's account of the photoelectric effects showed that light behaved as if it consisted of particle-like light quanta—later called "photons." Each photon has energy $E = hf$, where h is Planck's constant and f is the frequency of the light. Einstein himself pointed out that, since photons carry energy, this energy, while the photon moves at the speed of light, is equivalent to a certain amount of mass, according to his famous formula

$$E = mc^2.$$

The amount of mass is just

$$m = \frac{E}{c^2}.$$

Here c is the speed of light.

If a photon has energy, and the energy is equivalent to an amount of mass, does it also have momentum? The photoelectric effect did not tell anything about the momentum of a photon.

The magnitude of the momentum p of a body is defined as the product of its mass m and speed v in the same direction; $p = mv$. (See Chapter 5.) Replacing m with its energy equivalent E/c^2 gives

$$p = \frac{Ev}{c^2}.$$

Note that this equation is an expression for momentum p but that it contains no direct reference to mass. Now suppose this same equation is applied to the momentum of a photon of energy E. Since a photon moves at the speed of light, v would be replaced by the speed of light c to give

$$p = \frac{Ec}{c^2} = \frac{E}{c}.$$

Remember, $E = hf$ for a light quantum. If you substitute this expression for E in $p = E/c$, you get the expression

$$p = \frac{hf}{c}.$$

Using the wave relation that the speed equals the frequency times the wavelength, $c = f\lambda$, you can express the momentum of a photon as

$$\boxed{p = \frac{h}{\lambda}.}$$

FIGURE 15.1 Arthur H. Compton (1892–1962).

Does it make sense to define the momentum of a photon in this way? It would, if the definition helps in understanding experimental results. The first successful use of this definition was in the analysis of a now-famous phenomenon discovered by Arthur H. Compton, the *Compton effect*, which we describe below.

Consider a beam of light (or X rays) striking the atoms in a target, such as a thin sheet of metal. According to classical electromagnetic theory, the light will be scattered in various directions, but its frequency will not change. The absorption of light of a certain frequency by an atom may be followed by reemission of light of a different frequency. But if the light wave is simply *scattered*, then according to classical theory the frequency should not change.

According to quantum theory, however, light is made up of photons. According to relativity theory, photons have momentum. Therefore, Compton reasoned, in a collision between a photon and an atom, the law of conservation of momentum should apply. According to this law (Chapter 5), when a body of small mass collides with a massive object at rest, it simply bounces back or glances off. It experiences very little loss in speed and so very little change in energy. But if the masses of the two colliding objects are not very different, a significant amount of energy can be transferred in the collision. Compton calculated how much energy a photon should lose in a collision with an atom, if the photon's momentum is h/λ. He concluded that the change in energy is too small to observe if a photon simply bounces off an entire atom. But if a photon strikes an electron, which has a small mass, the photon should transfer a significant amount of energy to the electron.

In 1923, Compton was able to show that X rays did in fact behave like particles with momentum $p = h/\lambda$ when they collided with electrons. Compton measured the wavelength (or frequency) of the incident and the scattered X rays and thus was able to determine the X ray photon's change in momentum. By measuring separately the momentum of the scattered electron, he was able to verify that $p = h/\lambda$, using the law of conservation of momentum. For this work, Compton received the Nobel Prize in 1927.

Thus, Compton's experiment showed that a photon can be regarded as a particle with a definite momentum as well as energy. It also showed that collisions between photons and electrons obey the laws of conservation of momentum and energy.

As noted in Section 13.5, in the discussion of the photoelectric effect, light has particle-like properties. The momentum expression and the Compton effect gave additional evidence for this fact. To be sure, photons are not like ordinary particles, if only because photons do not exist at speeds other than that of light. (There can be no resting photons and, therefore, no rest mass for photons.) But in other ways, as in their scattering behav-

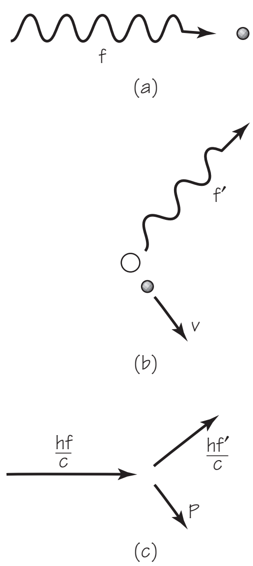

FIGURE 15.2 Compton's experiment: (a) X ray of frequency f approaches an electron; (b) X ray is scattered, leaving at lower frequency f', and electron recoils at velocity **v**; (c) the momentum before "collision" (hf/c) is equal to the vector sum of the momentum of both the scattered photon and the recoiling electron.

ior, photons do act much like particles of matter. For example, they have momentum as well as energy. Yet they also act like waves, having frequency and wavelength. In other words, electromagnetic radiation in some experiments exhibits behavior similar to what is thought of as particle behavior, and in other experiments, its behavior is similar to what is thought of as wave behavior. This pattern of behavior is often referred to as the *wave–particle dualism of radiation*. Is a photon a wave or a particle? The only answer, Bohr pointed out, is that it can *act* like either, depending on what is being done with it.

15.3 THE WAVE-LIKE BEHAVIOR OF PARTICLES

In 1923, the French physicist Louis de Broglie suggested that the wave–particle dualism that applies to electromagnetic radiation might also apply to electrons and other atomic particles. Perhaps, he said, the wave–particle dualism is a fundamental property of all quantum processes. If so, particles that were always thought of as material particles can, in some circumstances, act like waves. De Broglie sought an expression for the wavelength that might be associated with the wave-like behavior of an electron. He found such an expression by means of a simple argument.

The momentum of a photon of wavelength λ is $p = h/\lambda$. De Broglie thought that this relation might also apply to electrons with the momen-

LOUIS VICTOR DE BROGLIE

Prince Louis Victor de Broglie (1892–1987), whose ancestors served the French kings as far back as the time of Louis XIV, was educated at the Sorbonne in Paris. He proposed the idea of wave properties of electrons in his doctoral thesis.

FIGURE 15.3

tum $p = mv$. He therefore boldly suggested that the wavelength of an electron is

$$\lambda = \frac{h}{mv},$$

where m is the electron's mass and v its speed.

What does it mean to say that an electron has a wavelength equal to Planck's constant divided by its momentum? As before, if this statement is to have any physical meaning, it must be possible to test it by some kind of experiment. Some wave property of the electron must be measured. The first such property to be measured was *diffraction*.

The relationship $\lambda = h/mv$ indicates that the wavelengths associated with electrons will be very short, even for fairly slow electrons. An electron accelerated across a potential difference of only 100 V would have a wavelength of only 10^{-10} m. So small a wavelength would not give measurable diffraction effects on encountering even a microscopically small object (say, 10^{-5} m).

By 1920, it was well known that crystals have a regular lattice structure. The distance between rows of planes of atoms in a crystal is about 10^{-10} m. After de Broglie proposed that electrons have wave properties, several physicists suggested that the existence of electron waves might be shown by using crystals as diffraction gratings. Experiments begun in 1923 by C.J. Davisson and L.H. Germer in the United States yielded diffraction patterns similar to those obtained earlier for X rays (Section 13.7). See Figure 15.4.

The Davisson–Germer experiment showed two things. First, electrons *do* have wave properties, otherwise they could not display the diffraction pattern of waves. One may say that an electron moves along the path taken by the de Broglie wave that is associated with the electron. Second, electron wavelengths are correctly given by de Broglie's relation, $\lambda = h/mv$. These results were confirmed in 1927 when G.P. Thomson, the son of J.J. Thomson, directed an electron beam through thin gold foil. Thomson found a pattern like the one shown here. It resembles diffraction patterns produced by light beams going through thin slices of materials. By 1930, diffraction from crystals had been used to demonstrate the wave-like behavior even of helium atoms and hydrogen molecules. (It can be said that

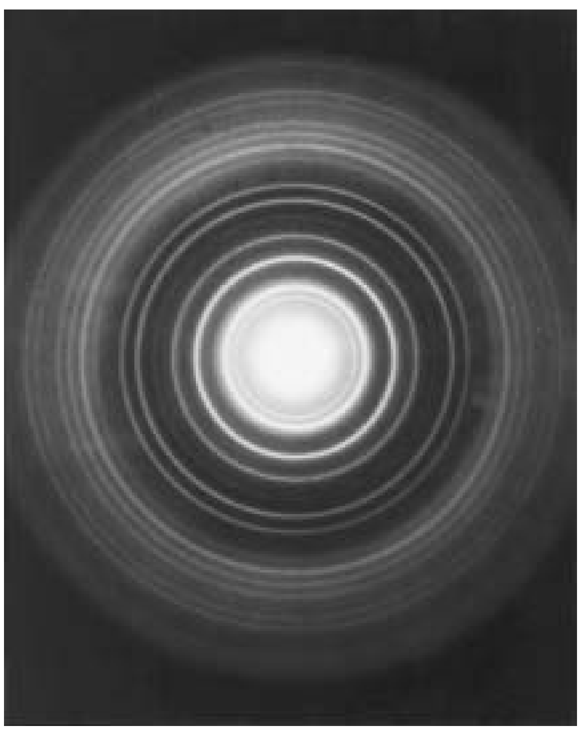

FIGURE 15.4 Diffraction pattern produced by directing a beam of electrons through polycrystalline aluminum (that is, many small crystals of aluminum oriented at random). With a similar pattern using gold foil, G.P. Thomson demonstrated the wave properties of electrons (28 years after their particle properties were first demonstrated by his father, J.J. Thomson).

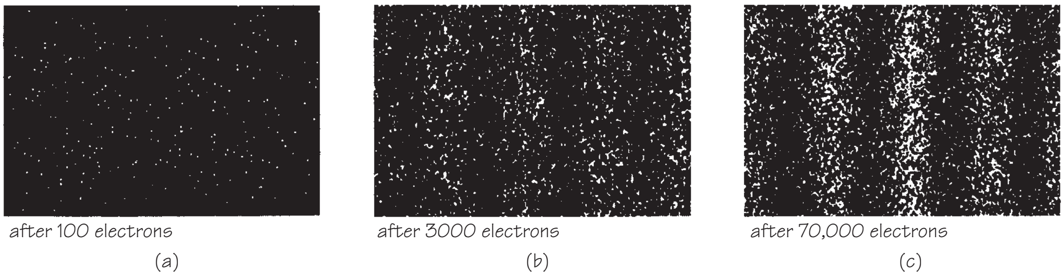

FIGURE 15.5 Various numbers of electrons passed through two slits to reach a screen forming an interference characteristic of waves.

J.J. Thomson discovered the electron *particle*, and his son showed it to be a *wave* as well.)

The experiments confirming de Broglie's hypothesis showed that the wave–particle dualism is a general property not only of radiation but also of matter. Scientists now customarily refer to electrons and photons as "particles" while recognizing that both have properties of waves as well.

Electron Waves and Atomic Structure

Bohr had postulated that the quantity mvr, which is called the "angular momentum" of the orbiting electron in the hydrogen atom, where r is the radius of the electron's orbit, m is the electrons mass, and v is its linear speed, can have only certain, quantized values. These quantized values help to define the stationary states. De Broglie's relation, $\lambda = h/mv$, has an interesting yet simple application that supports this postulate and sheds light on the existence of stationary states. Bohr assumed that mvr can have only the values

$$mvr = \frac{nh}{2\pi},$$

where, as before, $n = 1, 2, 3, \ldots$.

Now, suppose that an electron wave is somehow spread over an orbit of radius r so that, in some sense, it "occupies" the entire orbit. Can *standing waves* be set up as indicated, for example, in Figure 15.6? If so, the circumference ($2\pi r$) of the orbit must be equal in length to a whole number (n) of wavelengths, that is, to $n\lambda$. The mathematical expression for this condition of "fit" is

$$2\pi r = n\lambda.$$

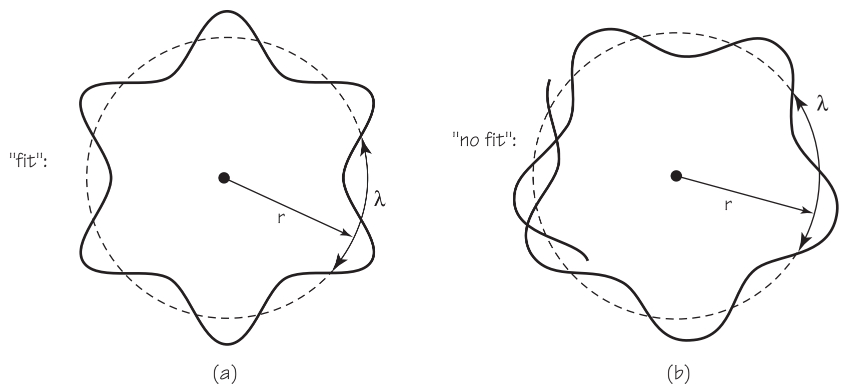

FIGURE 15.6 Only certain wavelengths will "fit" around a circle to form a standing wave.

Replacing λ by *h/mv* according to de Broglie's relation gives

$$2\pi r = \frac{nh}{mv}$$

or

$$mvr = \frac{nh}{2\pi}.$$

This is exactly equivalent to Bohr's quantization condition! The de Broglie relation for electron waves, and the idea that electrons have orbits that allow standing waves, allows us to *derive* the quantization of the electron orbits that Bohr had to *assume*.

The result obtained so far indicated that one could picture the electron in the hydrogen atom in two ways: as a *particle* moving in an orbit with a certain quantized value of *mvr*, or as a standing de Broglie-type *wave* occupying a certain region around the nucleus.

15.4 CONSTRUCTING QUANTUM MECHANICS

By the mid-1920s it was clear that "things" (electrons, atoms, molecules) long regarded as particles also show wave properties. This fact is the basis for the currently accepted theory of atomic structure. This theory, *quantum mechanics*, was introduced in 1925. Its foundations were developed very

FIGURE 15.7 Erwin Schrödinger (1887–1961) was born in Austria. He developed wave mechanics in 1926 and then fled from Germany in 1933 when Hitler and the Nazis came to power. From 1940 to 1956, he was professor of physics at the Dublin Institute for Advanced Studies.

rapidly during the next few years, primarily by Born, Heisenberg, Schrödinger, Bohr, Jordan, and Dirac. At first, the theory appeared in two different mathematical forms, proposed independently by Werner Heisenberg and Erwin Schrödinger. Heisenberg provided the basis for quantum mechanics emphasizing the particle aspect of quantum objects, while Schrödinger emphasized the wave aspect. Since Schrödinger's form of the theory is closer to the ideas of de Broglie (discussed in the previous section), it is often referred to as *wave mechanics*. Eventually Schrödinger proved that in fact these two forms of quantum mechanics are equivalent, that is, different ways of expressing the same relationships. Schrödinger's formulation is now predominant, although the symbols used in his equations are now interpreted differently from Schrödinger's original usage, as discussed presently.

Schrödinger sought to express the dual wave–particle nature of matter mathematically. Maxwell had formulated the electromagnetic theory of light in terms of a wave equation. Physicists were familiar with this theory and its applications. Schrödinger reasoned that the de Broglie waves associated with electrons could be described in a way analogous to the classical waves of light. Thus, there should be a wave equation that holds for matter waves, just as there is a wave equation for electromagnetic waves. This mathematical part of wave mechanics cannot be discussed adequately without using advanced mathematics, but the physical ideas involved require only a little mathematics and are essential to understanding modern physics. Therefore, the rest of this chapter will discuss some of the physical ideas of the theory so as to indicate that they are indeed reasonable. Some of the results of the theory and some of the significance of these results are also considered.

Schrödinger successfully derived an equation for the *matter waves* (de Broglie waves) that are associated with moving electrons. This equation, which has been named after him, defines the wave properties of electrons and also predicts particle-like behavior. The Schrödinger equation for an electron bound in an atom has a solution only when a constant in the equation has the whole-number values 1, 2, 3. . . . These numbers correspond to different energies. Thus, the Schrödinger equation predicts that only certain electron energies are possible in an atom. In the hydrogen atom,

FIGURE 15.8 P.A.M. Dirac (1902–1984), an English physicist, was one of the developers of modern quantum mechanics. In 1932, at the age of 30, Dirac was appointed Lucasian Professor of Mathematics at Cambridge University, the post held by Newton. Dirac is pictured here with Werner Heisenberg and Erwin Schrödinger while in Stockholm in 1933 on the occasion of the award of the Nobel Prize.

for example, the single electron can be in only those states for which the energy of the electron has the numerical values

$$E_n = \frac{k^2 2\pi^2 me^2}{n^2 h^2},$$

with n having only whole-number values. But these are just the energy values that are found experimentally, and just the ones given by the earlier Bohr theory! In addition, these states correspond to the picture of standing electron waves in the various stationary, as discussed in the previous section.

In Schrödinger's theory, these results follow directly from the mathematical formulation of the wave and particle nature of the electron. Bohr had to assume the existence of these stationary states at the start and make no assumptions about the allowable orbits. In Schrödinger's theory, however, the stationary states and their energies are *derived* from the theory. The new theory yields all the results of Bohr's theory, with none of the Bohr theory's inconsistent hypotheses. The new theory also accounts for certain experimental information for which Bohr's theory failed to account. For instance, it allows one to compute the intensity of a spectral line, which is understood as the probability of an electron changing from one energy state to another.

After the unification of Schrödinger wave mechanics with Heisenberg's formulation, quantum mechanics still contained the Schrödinger equation, but it not longer provided a physical model or visualizable picture of the atom. The planetary model of the atom has been given up and has not been replaced by another simple picture. There is now a highly successful *mathematical* model for the atom, but no easily visualized *physical* model. The concepts used to build quantum mechanics are more abstract than those of the Bohr theory. But the mathematical theory of quantum mechanics is much more powerful than the Bohr theory in predicting and explaining phenomena. Many problems that were previously unsolvable were rapidly solved with quantum mechanics (see the table on p. 673). Physicists have learned that the world of atoms, electrons, and photons cannot be thought of in the same mechanical terms as the world of everyday experience. Instead, the study of atoms presents some fascinating new concepts, such as those discussed below. What has been lost in easy visualizability is made up for by an increase in fundamental understanding about nature at the most fundamental level.

> What does it mean to "visualize" or "picture" something? One answer is that it means relating an abstract idea to something that you are familiar with from everyday life; for example, a particle is like a baseball or a marble. But why should there be anything from everyday life that is exactly like an electron or an atom?

The following table shows the rapid pace of development in accounting for some previously unsolvable problems and inexplicable phenomena following the formulation of quantum mechanics in 1926.

1926	Quantum mechanics of hydrogen atom	W. Pauli
1926	Quantum mechanics of helium atom	W. Heisenberg
1926	Hydrogen molecule	W. Heisenberg
1926	Effect of magnetic fields on spectral lines (Zeeman effect)	W. Heisenberg and P. Jordan
1927	Molecules in general	M. Born and J.R. Oppenheimer
1928	Magnetism (ferromagnetism)	W. Heisenberg
1928	Electron theory of metals	F. Bloch
1928	Relativistic quantum theory of electrons	P.A.M. Dirac

15.5 THE UNCERTAINTY PRINCIPLE

Up to this point, it has been assumed that any physical property of an object can be measured as accurately as necessary. To reach any desired degree of accuracy would require only a sufficiently precise instrument. Wave mechanics showed, however, that even in thought experiments with ideal instruments there are limits to the accuracy of measurements that can be achieved.

For example, think how you would go about measuring the positions and velocity of a car moving slowly along a driveway. You could mark the position of the front end of the car at a given instant by making a scratch on the ground. At the same time, you could start a stopwatch. Then you could run to the end of the driveway, where you have previously placed another mark. At the instant when the front of the car reaches this point, you stop the watch. You then measure the distance between the marks and get the average speed of the car by dividing the distance traveled by the time elapsed. Since you know the direction of the car's motion, you know the average velocity. Thus, you know that at the moment the car reached the second mark it was at a certain distance from its starting point and had traveled at a certain average velocity. By going to smaller and smaller intervals, you could also get the instantaneous velocity at any point along its path.

How did you get the needed information? You located the car by sunlight that was bounced off the front end into your eyes. The light permitted you to see when the car reached a mark on the ground. To get the average speed, you had to locate twice where the front end was.

But suppose that you had decided to use reflected radio waves instead of visible light. At 1000 kHz, a typical value for radio signals, the wavelength is 300 m. This wavelength is very much greater than the dimensions of the car. Thus, it would be impossible to locate the position of the car with any accuracy. The wave would reflect from the car ("scatter" is a better term) in all directions. It would also sweep around any human-sized device you may wish to use to detect the wave direction. The wavelength has to be comparable with or smaller than the dimensions of the object before the object can be located well.

Radar uses wavelengths from about 0.1 cm to about 3 cm, so a radar apparatus could be used instead of sunlight. But even radar would leave uncertainties as large as several centimeters in the two measurements of position. The wavelength of visible light is less than 10^{-6} m. For visible light, then, you could design instruments that would locate the position of the car to an accuracy of a few thousandths of a millimeter.

Now think of an electron moving along an evacuated tube. You will try to measure the position and speed of the electron. But you must change your method of measurement. The electron is so small that you cannot locate its position by using ordinary visible light. (The wavelength of visible light, small as it is, is still at least 10^4 times greater than the diameter of an atom.)

FIGURE 15.9 Werner Heisenberg (1901–1976), a German physicist, was one of the developers of modern quantum mechanics (at the age of 23). He was the first to state the uncertainty principle. After the discovery of the neutron in 1932, he proposed the proton-neutron theory of nuclear structure.

You are attempting to locate the electron within a tiny region, say the size of an atom, about 10^{-10} m across. So you need a light beam whose wavelength is about 10^{-10} m or smaller. But a photon of such a short wavelength λ (and high frequency f) has very great momentum (h/λ) and energy (hf). Recalling Compton's work (Section 15.2), you know that such a photon will give the electron a strong kick when it is scattered by the electron. As a result, the velocity of the electron will be greatly changed, into a new and unknown direction. (This is a new problem, one you did not even think about when measuring the position of the car!) Therefore, when you receive the scattered photon, you can deduce from its direction where the electron *once was*; in this sense you can "locate" the electron. But in the process you have changed the velocity, hence the momentum, of the electron (in both magnitude and direction). In short, the more accurately you locate the electron by using photons of shorter wavelength, the less accurately you can know its momentum. You could try to disturb the electron less by using less energetic photons. But because light exists in quanta of energy hf, a *lower-energy* photon will have a *longer* wavelength. This would create greater uncertainty about the electron's *position*! In other words:

> It is impossible to measure both the position and the momentum of a subatomic particle, in the same instant to unlimited accuracy. The more accurate is the measurement of the momentum, the less accurate is the measurement of the position in that instant, and vice versa.

This conclusion is expressed in the *uncertainty principle*, first stated by Werner Heisenberg in 1927. The uncertainty principle can be expressed quantitatively in two simple mathematical expressions, known as *uncertainty relations*, which, as indicated by the above example, are necessary conclusions drawn from experimental facts about measurements involving quantum objects.

In this example, let Δx represent the uncertainty in the measurement of the position of the object, and let Δp_x be the uncertainty in the measurement of the momentum of the object in the x direction at the same instant. Heisenberg's principle says that the product of these two uncertainties must be equal to, or greater than, Planck's constant divided by 4π:

$$\Delta x \cdot \Delta p_x \geq \frac{h}{4\pi}.$$

There are similar relations for the momenta in the y and z directions and the uncertainties in the measurements of the position along the y and z coordinates.

This uncertainty relation says that if we use a short-wavelength photon in an attempt to measure the position of an electron to very high accuracy, so that Δx is very small, then the uncertainty in the momentum measurement Δp must be at least $h/4\pi\Delta x$. This means that as Δx gets smaller, Δp has to get larger. Again one can see this also from the Compton effect; the electron would bounce away faster the shorter the wavelength (the higher the energy) of the measuring photon. In fact, if we measure the position so accurately that there is no uncertainty at all in the position, then Δx would be zero. But to do this we would have been forced to use a photon whose wavelength λ was zero! Such a photon has infinite energy! In that case, the uncertainty in the momentum of the electron would be infinite, or undefined.

On the other hand, if we allowed the uncertainty in the position measurement to become very large, then the uncertainty in the momentum measurement would become very small, since the photon would have long wavelength (low momentum). If Δx became so large that it was infinite, or undefined, then Δp would become zero. We could measure the momentum in that instant with absolute precision. But we cannot measure both the position and the momentum to absolute precision at the *same* time. Instead, the uncertainty relation forces us into a trade-off. When the precision of one variable goes up, the other must go down, and vice versa.

Similar considerations result in a second uncertainty relation involving the variables time and energy. Let Δt represent the uncertainty in the measurement of time, and ΔE the uncertainty in the measurement of the energy of a quantum object at the same instant. Heisenberg's uncertainty principle expresses the uncertainty relation for these two uncertainties

$$\Delta t \cdot \Delta E \geq \frac{h}{4\pi}.$$

Similarly to the above relationship, this relationship states:

> It is impossible to measure both the time and the energy of a quantum object in the same instant to unlimited accuracy. The more accurate is the measurement of the energy, the less accurate is the measurement of the time in that instant, and vice versa.

15.6 ORIGINS AND A CONSEQUENCE OF THE UNCERTAINTY PRINCIPLE

The uncertainty principle and the two resulting uncertainty relations hold for any object—even our earlier experiment on the car. But the limitations imposed by the uncertainty principle have no practical consequence for such massive objects as cars or baseballs moving at everyday speeds. This is because the amounts of uncertainty involved are too small to be noticed. It is only on the atomic scale that the limitations become evident and important.

Whose Fault Is It?

It is important to understand that the uncertainties mandated by the uncertainty principle are not the fault of the experimenter or of the instruments we use. We can *never* build instruments to get around the reciprocal uncertainties in measurements imposed by the uncertainty relations. This is because Heisenberg's uncertainty relations are a direct consequence of quantum mechanics and the wave–particle duality, and they will remain valid as long as quantum mechanics remains an acceptable theory. In fact, the uncertainty relations may be even more fundamental than quantum mechanics, for these relations seem to be connected with the very existence of the quantum itself. You have seen the role that Planck's constant h has played in defining the light quantum and in accounting for the stationary states in the Bohr atom. In addition, the constant h appears in both of the basic equations for the energy and the momentum of a photon, $E = hf$ and $p = h/\lambda$, and in many other quantum equations as well. It also appears in the two uncertainty relations. If h were 0, the quantum of energy would be zero, so there would be no light quanta, only continuous waves. The momentum of a photon would also be zero, and the uncertainty relations would read

$$\Delta x \cdot \Delta p_x = 0,$$

$$\Delta t \cdot \Delta E = 0.$$

In this case there would be no reciprocal uncertainties in position and momentum, time and energy. We could measure simultaneously the wave and particle features of quantum objects without any problem. But Planck's constant is *not* zero (although it is very small), the quantum does exist, we are faced with the wave–particle duality, quantum mechanics is still an accepted theory, and nature is so arranged as to limit the precision of our measurements of fundamental quantities at the most fundamental level.

The Sizes of Atoms

On the atomic scale one of the main uses made of the uncertainty principle is in general arguments in atomic theory rather than in particular numerical problems. For instance, the uncertainty principle helps to answer a long-standing fundamental question: Why do atoms have the sizes that they do? As you saw previously (Chapter 14), atoms are actually made up mostly of empty space. At the center of each atom is a very tiny nucleus, which carries all of the positive charge and nearly all of the mass of the atom. Surrounding the nucleus are a number of electrons, equal to the positive charge of the nucleus. The electrons are arranged on various quantum orbits. The lowest one is called the "ground state." But even the orbit of the ground state is still far away from the nucleus. In most atoms, the radius of the ground state is about 10^{-8} cm, while the radius of the nucleus is only about 10^{-12} cm. This means that a nucleus occupies only a tiny fraction of the space inside an atom; the rest is empty (except for a few point-sized electrons.)

As you saw with Rutherford's model, the atom should collapse into the nucleus, because the electrons should radiate away their energy and spiral inward. Bohr attempted to account for why this does not happen by postulating the existence of stationary quantum states, while quantum mechanics associated such states with standing waves, the ground state having the smallest standing electron wave in that orbit. But it is the uncertainty principle that explains why we can't have any lower states, and why the negative electrons cannot exist within or on the positive nucleus. A simple application of the uncertainty relation for position and momentum (see Quantitative Discovery Question 3) shows that if an electron is confined to a space of 10^{-8} cm, the size of an average atom in centimeters, then the uncertainty in its speed is less than the speed of light. But if it is confined to a much smaller space, or even down to the size of the nucleus, the uncertainty in its speed would exceed the speed of light, which is 3×10^{10} cm/s. As you know from relativity theory, no material particle may exceed the speed of light. So the space inside the atom between the nucleus and the first quantum state must be left empty.

Why Aren't Atoms Any Larger?

To increase the size of an atom, we would have to bring electrons into much higher quantum states. Aside from requiring the input of a lot of energy, the higher states are not evenly spaced but become farther and farther apart. Thus, the likelihood increases greatly that the electrons in these higher states can escape the atom and become free, so such an atom would not exist for long, which means that in practice most atoms one can study are about 10^{-8} cm in size. You will see in the next chapter that the fact that atoms do have a size of about 10^{-8} cm helps to account for many of the properties of matter that we see around us.

15.7 THE PROBABILITY INTERPRETATION

The wave–particle dualism is also a fundamental aspect of quantum mechanics. In order to explore this dualism further, it is necessary to review some ideas of probability. In some situations, no single event can be predicted with certainty. But it may still be possible to predict the *statistical probabilities* of certain events. For example, on a holiday weekend during which perhaps 25 million cars are on the road in the United States, statisticians, basing themselves on past experience, predict that about 400 people will be killed in car accidents. It is of course not known which cars in which of the 50 states will be involved in the accidents. But the *average* behavior is still quite accurately predictable.

Along somewhat similar lines, physicists think about the behavior of photons and material particles. As you have seen, there are basic limitations on the ability to describe the behavior of an individual particle. But the laws of physics often make it possible to describe the behavior of large collections of particles with good accuracy (as was the case in the kinetic theory of gasses, Chapter 7). Schrödinger's equation for the waves associated with quantum particles gives the *probabilities* for finding the particles at a given place and a given time; it does not give the precise behavior of an individual particle.

In order to see how quantum probability works, consider the situation of a star being photographed through a telescope. As you have already seen, for example, in the case of waves striking a barrier in which there is a single opening about the size of the wavelength (see Chapter 8), the image formed on a screen behind the opening is not a precise point. Rather, it is a *diffraction pattern*, a central spot with a series of progressively fainter circular rings.

The image of a star on the photographic film in a telescope would be a similar pattern. Imagine now that you wish to photograph a very faint star. If the energy in light rays were not quantized, it would spread continuously over ever-expanding wave fronts. Thus, you would expect the image of a very faint star to be exactly the same as that of a much brighter star, except that the intensity of light would be less over the whole pattern. However, the energy of light *is* quantized; it exists in separate quanta, "photons," of a definite energy, which obey Schrödinger's equation. A photon striking a photographic emulsion produces a chemical change in the film at a single location, not all over the image area. That location, however, is not predictable in advance. All that we can predict is the *probability* that the photon might arrive at that location.

If the star is very remote, only a few photons per second may arrive at

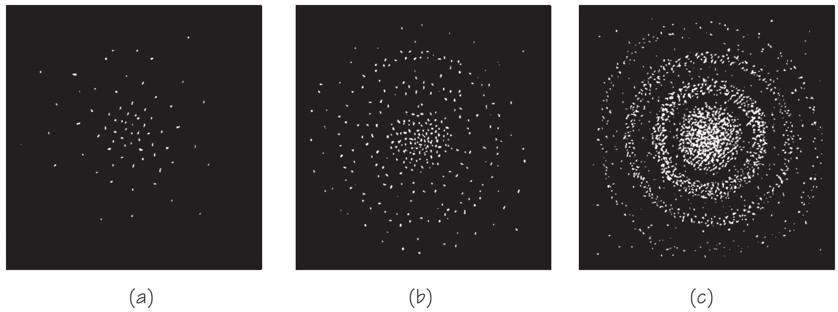

FIGURE 15.10 These sketches represent successive stages of a greatly enlarged image of a distant star on a photographic plate showing the impacts of individual photons.

the film. The effect on the film after a very short period of exposure would be something like the pattern in Figure 15.10(a). As the exposure continues, the effect on the film would begin to look like (b). Finally, after many photons have arrived from the star, a pattern like (c) would be produced, just like the image produced by a much brighter star with a much shorter exposure. (Compare with Figure 15.5.)

For huge numbers of *particle-like* photons, the overall distribution they form is very well described by the distribution expected on the basis of the *wave* intensity of light. For small numbers of quanta, the wave intensity is not very useful for predicting where the photons will go. One might expect them to go mostly to the "high-intensity" parts of the image, but one cannot predict exactly where for an individual photon. These facts fit together beautifully if you consider the wave intensity at a location to indicate the *probability* of a photon going there!

A similar connection can be made for de Broglie waves and particles of matter. For this purpose, rather than considering a diffraction pattern formed by an electron beam, consider an electron wave that is confined to a particular region in space. An example is the de Broglie wave associated with the electron in a hydrogen atom, which is spread out all over the atom. Another example is the de Broglie wave of an electron in a good conductor of electricity. The wave's amplitude at some location represents the probability of the electron being there, if a measurement of the electron's location were to be performed.

According to quantum mechanics, the hydrogen atom does not consist of a localized negative particle (an electron) moving around a nucleus as in the Bohr model. Indeed, the theory does not provide any fixed, easily visualizable picture of the hydrogen atom. A description of the probability distribution is the closest thing to a picture that quantum mechanics provides.

As discussed in Chapter 7 in connection with kinetic theory and disorder, it is easy to predict the average behavior of very large numbers of particles, even though nothing is known about the behavior of any single one of them. Unlike kinetic theory, however, the use of probabilities in quantum mechanics is not for convenience, but seems to be an intrinsic necessity. There is no other way to deal with quantum mechanics. The theory is not really concerned with the position of any individual electron in any individual atom, but rather gives a mathematical representation that can be used to predict interactions with particles, fields, and radiation. For example, it can be used to calculate the probability that hydrogen will emit light of a particular wavelength. The intensity and wavelength of light emitted by a large number of hydrogen atoms can then be compared with these calculations. Comparisons such as these have shown that the theory agrees with experiment.

In most cases, atomic physics deals with the average behavior of many atomic particles. The laws governing this average behavior are those of quantum mechanics. The waves can be considered waves whose amplitudes are a measure of probability. The information (concerning the probability with which a particle will reach some position at a given time) travels through space in probability waves. These waves can interfere with each other in exactly the same way that water waves do. For example, think of a beam of electrons passing through two slits. In such an experiment one can consider the electrons to be waves, and one can compute their interference pattern. The interference pattern is actually a probability pattern that provides the probabilities that individual electrons will arrive at different locations behind the slits. Large constructive interference at a location indicates high probability that electrons will arrive there; large destructive wave interference indicates low or vanishing probability. We cannot say where an individual electron will end up after passing through the slits; all we can know, according to quantum mechanics, is the probability of landing at each location. However, after the passage of many electrons, the statistical buildup of particle-like electrons will provide the familiar interference pattern that we expect to see for waves. As Max Born, the primary founder of the probability interpretation, wrote in 1926:

> The motion of particles conforms to the laws of probability, but the probability itself is propagated in accordance with the law of causality.

15.8 THE COMPLEMENTARITY PRINCIPLE

Quantum mechanics was founded upon the existence of the wave–particle dualism of light and matter, and the enormous success of quantum mechanics, including the probability interpretation, seems to reinforce the importance of this dualism. But how can a particle be thought of as "really" having wave properties? And how can a wave be thought of as "really" having particle properties? One could build a consistent quantum mechanics upon the idea that a light beam or an electron can be described simultaneously by the incompatible wave and particle concepts.

In 1927, Niels Bohr realized that the word "simultaneously" provided the key to a consistent account. He realized that our models, or pictures, of matter and light are based upon their behavior in various experiments in our laboratories. In some experiments, such as the photoelectric effect or the Compton effect, light behaves as if it consists of particles; in other experiments, such as the double-slit experiment, light behaves as if it consists of waves. Similarly, in experiments such as J.J. Thomson's cathode-ray studies, electrons behave as if they are particles; in other experiments, such as his son's diffraction studies, electrons behave as if they are waves. But light and electrons never behave *simultaneously* as if they consist of both particles and waves. In each specific experiment they behave *either* as particles *or* as waves, but never as both.

This suggested to Bohr that the particle and wave descriptions of light and of matter are both necessary even though they are logically incompatible with each other. They must be regarded as being "complementary" to each other—that is, like two different sides of the same coin. This led Bohr to formulate what is called the *Principle of Complementarity*:

> The wave and particle models are both required for a complete description of matter and of electromagnetic radiation. Since these two models are mutually exclusive, they cannot be used simultaneously. Each experiment, or the experimenter who designs the experiment, selects one or the other description as the proper description for that experiment.

Bohr showed that this principle is a fundamental consequence of quantum mechanics. He handled the wave–particle duality, not by resolving it in favor of either waves or particles, but by absorbing it into the foundations of quantum physics. Like the Bohr atom, it was another bold initiative toward the formulation of a new theory, even though this required contradictions with classical physics.

FIGURE 15.11 Max Born (1882–1969) was born in Germany but left for England in 1933 when Hitler and the Nazis came to power. Born was largely responsible for introducing the statistical interpretation of wave mechanics.

It is important to understand what the complementarity principle really means. By accepting the wave–particle duality as a fact of nature, Bohr was saying that light and electrons (or other objects) encompass potentially the properties of both particles and waves—until they are observed, at which point they behave as *if* they are either one or the other, depending upon the experiment and the experimenter's choice. This was a profound statement, for it meant that what we observe in our experiments is not the way nature "really is" when we are not observing it. In fact, nature does not favor any specific model when we are not observing it; rather, it is a mixture of the many possibilities that it could be until we finally do observe it! By setting up an experiment, *we* select the model that nature will exhibit, and *we* decide how photons and electrons and protons and even baseballs (if they move fast enough) are going to behave—either as particles or as waves.

In other words, according to Bohr, *the experimenter becomes part of the experiment!* In so doing, the experimenter interacts with nature, so that we can never observe all aspects of nature "as she really is" by herself. In fact, that phrase, while so appealing, has no operational meaning. Instead, we should say we can know only the part of nature that is revealed by our experiments. (This is no invitation to mysticism. After all, we know even about a good friend only through a patchwork of repeated encounters and discussions, in many different circumstances.) The consequence of this fact, for events at the quantum level, said Bohr, is the uncertainty principle, which places a quantitative limitation upon what we can learn about nature in any given interaction; and the consequence of this limitation is that we must accept the probability interpretation of individual quantum processes. For this reason, the uncertainty principle is often also called the principle of indeterminacy. There is no way of getting around these limitations, according to Bohr, as long as quantum mechanics remains a valid theory.

Such ideas, of course, are totally at odds with how we usually think of nature. We usually think of nature as existing completely independently of us and possessing a definite reality and behavior even when we are not observing it. For instance, you assume that the world outside of the place where you are now sitting still exists pretty much as it was when you last observed it. Certainly this is the way nature always behaves in our everyday experience, and this view is a fundamental assumption of classical physics. It even has a philosophical name; it is called "realism," and for phenomena and objects in the range of ordinary experience is perfectly appropriate. But, as Bohr so often emphasized, we have to be prepared to expect that the quantum world will not be anything like the everyday world in which we live. Max Born, one of the founders of quantum mechanics, has written:

> The ultimate origin of the difficulty lies in the fact (or philosophical principle) that we are compelled to use the words of common language when we wish to describe a phenomenon, not by logical or mathematical analysis, but by a picture appealing to the imagination. Common language has grown by everyday experience and can never surpass these limits. Classical physics has restricted itself to the use of concepts of this kind; by analyzing visible motions it has developed two ways of representing them by elementary processes: moving particles and waves. There is no other way of giving a pictorial description of motions—we have to apply it even in the region of atomic processes, where classical physics breaks down.

Bohr's complementarity principle, Heisenberg's uncertainty principle, and Born's probability interpretation together form a logically consistent interpretation of the meaning of quantum mechanics. Since this interpretation was developed largely in Bohr's institute at the University of Copenhagen, it has been called the *Copenhagen Interpretation* of quantum mechanics. The results of this interpretation have profound scientific and philosophical consequences that have been studied and debated to this day.

15.9 SOME REACTIONS

The idea that the solution of Schrödinger's equation is a wave that represents, not a physical wave, but the probability of finding the associated particle in some specific condition of motion has had great success. In fact, every experiment devised so far to test this interpretation has confirmed these results. Yet many scientists still find it hard to accept the idea that it is impossible to know exactly what any one particle is doing. The most prominent of such disbelievers was Einstein. In a letter to his friend and colleague Max Born, written in 1926, he remarked:

> The quantum mechanics is very imposing. But an inner voice tells me that it is still not the final truth. The theory yields much, but it hardly brings us nearer to the secret of the Old One. In any case, I am convinced that He does not play dice.

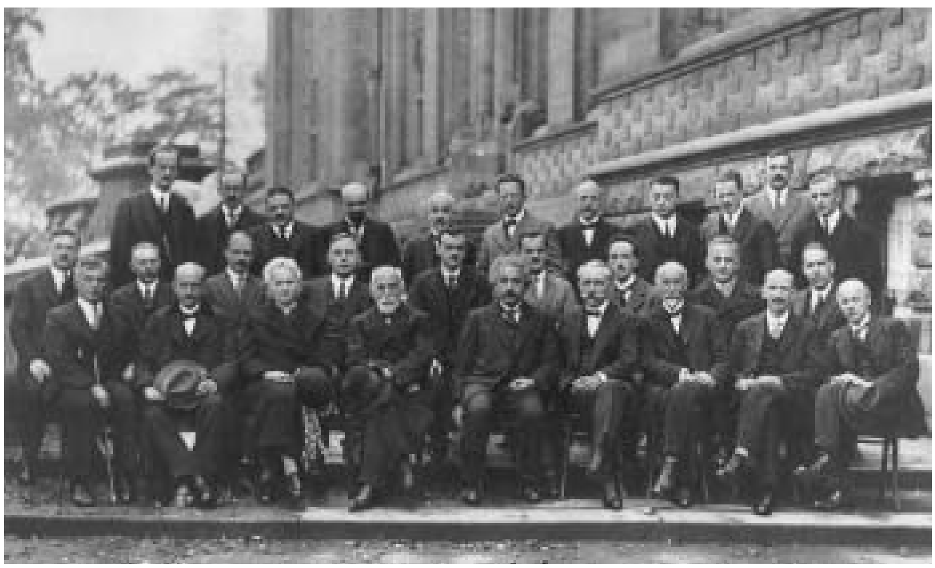

FIGURE 15.12 Photograph of participants in the Fifth Solvay Congress, Brussels, 1927, a veritable "Who's Who" of physics in the first half of the twentieth century. Back row, from left: Auguste Piccard, E. Henriot, Paul Ehrenfest, E. Herzen, T. de Donder, Erwin Schrödinger, E. Verschaffelt, Wolfgang Pauli, Werner Heisenberg, Ralph Fowler, Leon Brillouin. Middle row, from left: Peter Debye, Martin Knudsen, William L. Bragg, H.A. Kramers, Paul Dirac, Arthur Holly Compton, Louis de Broglie, Max Born, Niels Bohr. Front row, from left: Irving Langmuir, Max Planck, Marie Curie, H.A. Lorentz, Albert Einstein, Paul Langevin, Charles Guye, Charles Wilson, Owen Richardson.

Thus, Einstein agreed with the usefulness and success of quantum mechanics, but he refused to accept probability-based laws as the final level of explanation in physics. The remark about not believing that God played dice (an expression he used many times later) expressed Einstein's faith that more basic, deterministic laws are yet to be found. By this he meant that if all the conditions of an isolated system are known and the laws describing the interactions are known, then it should be possible to predict precisely, according to "strict causality," what will happen next, without any need for probability.

Some scientists agreed with Einstein, but all scientists do agree that, as a theory, quantum mechanics does work in practice. It gives the right answers to many questions in physics; it unifies ideas and occurrences that were once unconnected; it has produced many new experiments and new concepts; and it has opened the door to many technological advances, from transistors, microprocessors, and superconductors, to lasers and the latest medical-imaging techniques.

On the other hand, there is still vigorous argument about the basic significance of quantum mechanics. It yields probability functions, not precise trajectories. Some scientists see in this aspect of the theory an important indication of the nature of the world. For other scientists, the same fact indicates that quantum mechanics is still incomplete. Some in this second group are trying to develop a more fundamental, nonstatistical theory. For such a theory, the present quantum mechanics is only a special case. As in other fields of physics, the greatest discoveries here may be those yet to be made.

SOME NEW IDEAS AND CONCEPTS

complementarity
Compton effect
Copenhagen interpretation
de Broglie wave
double-slit experiment
photon
probability interpretation
quantum mechanics
Schrödinger equation
uncertainty principle
wave mechanics
wave–particle dualism

SOME IMPORTANT EQUATIONS

$$E = hf,$$

$$m = \frac{E}{c^2},$$

$$p = \frac{h}{\lambda},$$

$$\lambda = \frac{h}{mv},$$

$$\Delta x \cdot \Delta x \geq h/4\pi,$$

$$\Delta t \cdot \Delta E \geq h/4\pi.$$

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STUDY GUIDE QUESTIONS

15.1 The Quantum

1. Why did the existence of the quantum of energy pose a major problem for physicists?

15.2 The Particle-Like Behavior of Light

1. Why was momentum conservation not considered in the discussion of the photoelectric effect?
2. How does the momentum of a photon depend on the frequency of the light?
3. How could you figure out how much mass a photon carries?
4. Write down the steps leading the expression for the momentum, $p = h/\lambda$.
5. What did Compton do, and what did the experiment prove?
6. What is the best answer to the question "Is the photon a particle or a wave?"

15.3 The Wave-Like Behavior of Particles

1. How did de Broglie obtain the relation $\lambda = h/mv$ for electrons?
2. Why were crystals used to get diffraction patterns of electrons?
3. How can electron waves be used to explain the origin of the stationary states that Bohr assumed in his theory?
4. What two pictures of the atom were possible as a result of the discovery of electron waves?
5. How would you obtain the wavelength of a moving electron?
6. In what way is nature symmetric in the wave–particle duality?

15.4 Constructing Quantum Mechanics

1. The set of energy states of hydrogen could be derived from Bohr's postulate that $mvr = nh/2\pi$. In what respect was the derivation from Schrödinger's equation better?
2. Quantum (or wave) mechanics has had great success. What is its drawback for those trained to think in terms of physical models?

15.5 The Uncertainty Principle

1. What is the "uncertainty" of a measurement?
2. If photons used in finding the momentum of an electron disturb the electron too much, why cannot the observation be improved by using less energetic photons?

3. If the wavelength of light used to locate a particle is too long, why cannot the location be found more precisely by using light of shorter wavelength?
4. If you measured the position of an electron with very small uncertainty, what consequence would this have for the measurement of its momentum? Explain.
5. What effect does the greater-than-or-equal-to sign, ≥, have on the relationship between the two uncertainties?

15.6 Origins and a Consequence of the Uncertainty Principle

1. Whose fault is it that we can't get rid of the uncertainty in our measurements?
2. Why can't atoms have much larger sizes?
3. Why can't electrons exist inside the nucleus?

15.7 The Probability Interpretation

1. In wave terms, the bright lines of a diffraction pattern are regions where there is a high field intensity produced by constructive interference. What does the probability interpretation say about the bright lines of a diffraction pattern?
2. Quantum mechanics can predict only probabilities for the behavior of any one particle. How, then, can it predict many phenomena, for example, half-lives and diffraction patterns, with great certainty?
3. Individual photons are sent onto the double-slits of Young's experiment. They hit the photographic plate in random fashion as quantum mechanics requires. If that is the case, then how is the wave interference pattern formed?
4. Explain in your own words what Born is saying in the quotation.
5. How does the probability interpretation in quantum mechanics differ from the probability interpretation of entropy in the kinetic theory of gases?
6. How can nature be fundamentally random yet at the same time we can predict with great accuracy the interference pattern that is formed in Young's experiment?

15.8 The Complementarity Principle

1. What did Bohr realize when he examined experiments on particles and waves?
2. What does the word "complementary" mean?
3. State the principle of complementarity in your own words.
4. How does the experimenter become part of the experiment?
5. How does this contradict the idea of "realism"?
6. How did Bohr regard the strange behavior of the quantum world?
7. Why can't we know nature as it really is, according to quantum mechanics?

15.9 Some Reactions

1. What was Einstein's objection to the probability interpretation?
2. What did Einstein mean when he said God does not play dice?

DISCOVERY QUESTIONS

1. What would happen to the wave–particle dualism if Planck's constant $h = 0$? What would happen to the uncertainty principle if Planck's constant $h = 0$?
2. Why can't we measure both the position and the time simultaneously with absolutely no uncertainty?
3. How do you think it was possible for Einstein to be a primary founder of quantum theory, yet object to quantum mechanics?
4. Review some of the new ideas and concepts in this chapter. Which ones do you find to be the most startling, and the most disturbing. Evaluate why you find them to be startling and disturbing.
5. Niels Bohr thought the complementarity principle applies also to events in our lives—for example, that one loves one's child and yet has to punish him or her for bad behavior. Can you think of other examples where a "complementarity principle" may apply to ordinary life?
6. What explanation would you offer for the fact that the wave aspect of light was shown to be valid before the particle aspect was demonstrated?
7. Suppose that the only way you could obtain information about the world was by throwing rubber balls at the objects around you and measuring their speeds and directions of rebound. What kind of objects would you be unable to learn about? How does this question relate to this chapter?
8. Some writers have claimed that the uncertainty principle proves that there is free will. Do you think this extrapolation from atomic phenomena to the world of living beings is valid?
9. A physicist has written:

   > It is enough that quantum mechanics predicts the average value of observable quantities correctly. It is not really essential that the mathematical symbols and processes correspond to some intelligible physical picture of the atomic world.

   Do you regard such a statement as acceptable? Give your reasons.
10. Previous chapters have discussed the behavior of large-scale "classical particles" (e.g., tennis balls) and "classical waves" (e.g., sound waves). Such particles and waves in most cases can be described without any use of ideas such as the quantum of energy or the de Broglie matter wave. Does this mean that there is one sort of physics ("classical physics") for the phenomena of the large-scale world and quite a different physics ("quantum physics") for the phenomena of the atomic world? Or does it mean that quantum physics really applies to all phenomena but is no different from classical physics when applied to large-scale particles and waves? What arguments or examples would you use to defend your answer?
11. Some writers have declared that the impossibility of finding classical causality at the level of quantum objects proves "science cannot really know nature." Some also claim that it coincides with aspects of Eastern religions. What would be your responses?

Quantitative

1. Most professional pitchers can throw a baseball at 100 mi/hr (62 km/hr). What would be the de Broglie wavelength of a ball of mass 147 g at that speed? Could this wavelength be detected by crystal diffraction?
2. What is your de Broglie wavelength when you walk at a brisk pace of 4 mi/hr? Why have you not experienced the wave side of yourself?
3. (a) Once it was thought that β rays from the nucleus are electrons that were originally present inside the nucleus. According to the uncertainty principle, what would be the approximate range of speeds (uncertainty of speed) of an electron confined to a nucleus of size 10^{-14} m? The rest mass of an electron is about 9.1×10^{-31} kg; assume it is not subject to relativistic mass increase. How does the result of your calculation compare with the speed of light? What does this tell about the old idea of the presence of electrons in the nucleus?
   (b) What would be the approximate uncertainty in the speed of an electron confined to the first Bohr orbit of a hydrogen atom, where $r = 5.29 \times 10^{-9}$ cm?
   (c) According to the uncertainty principle, what would be the approximate speed of a proton confined to a nucleus of size 10^{-14} m? Neglect relativistic effects. (The rest mass of a proton is about 1.6×10^{-27} kg.) How does this compare with the speed of light?
   (d) On the basis of your results, why can a proton exist in the nucleus if an electron cannot?
4. An electron is fired from an "electron gun" in the CRT of a computer monitor. The electron is aimed at a blue-fluorescent subpixel on the screen of 10^{-6} m in width. What is the minimum uncertainty in the electron's momentum in the horizontal direction along the screen?
5. Calculate the momentum of a photon of wavelength 400×10^{-9} m. How fast would an electron have to move in order to have the same momentum? What would be the wavelength of an electron moving at that speed?